Assignment 2

You must submit it electronically to ELMS. This is a group assignment. Every group only needs to submit one solution. Group members get the same credit for the group submission.

This assignment is 7% in your total points. For the simplicity of the grading, the total points for the assignment is 70.

Problem 1 [10 pts]. Prove that if \( P = \text{PSPACE} \), then \( \text{EXP-TIME} = \text{EXP-SPACE} \). (Hint: use the padding technique.)

Problem 2 [15 pts]. Show that the following language is undecidable:

\[ \{ <M> : M \text{ is a machine that runs in } 100n^2 + 200, \text{ where } n \text{ is the input size} \} \]

(Hint: use the diagonalization method or a reduction from the \text{HALT} problem.)

Problem 3 [25 pts]. Recall that the language \( \text{SAT}_H \) is defined by

\[ \text{SAT}_H = \left\{ \psi 01^n H(n) : \psi \in \text{SAT}, \text{ and } n = |\psi| \right\}, \]

where \( H(n) \) is the smallest number \( i < \log \log n \) such that for every \( x \in \{0,1\}^* \) with \( |x| \leq \log n \), \( M_i \) outputs \( \text{SAT}_H \) within \( i|x|^i \) steps. If there is no such number \( i \) then \( H(n) = \log \log n \). (\( M_i \) denotes the \( i \)th Turing machine.)

- (10 pts) Prove that the function \( H \) defined is computable in polynomial time.
- (15 pts) Prove that if \( \text{SAT}_H \) is \text{NP-complete}, then \( \text{SAT} \) is in \( \text{P} \).

(Hint: Arora-Barak page 532. There is a scanned version posted in Piazza.)

Problem 4 [20 pts]. Show that 2SAT is in \( \text{NL} \). (Hint: reduce 2SAT to \( \text{PATH} \).)