Assignment 2

You must submit it electronically to ELMS. This is a group assignment. Every group only needs to submit one solution. Group members get the same credit for the group submission.

This assignment is 7% in your total points. For the simplicity of the grading, the total points for the assignment is 70.

Problem 1 [10 pts]. Prove that if P=PSPACE, then EXP-TIME=EXP-SPACE. (Hint: use the padding technique.)

Problem 2 [15 pts]. Show that the following language is undecidable:

\{ <M> : M is a machine that runs in 100n^2 + 200, where n is the input size \}.

(Hint: use the diagonalization method.)

Problem 3 [25 pts]. Recall that the language SAT_H is defined by

\[
\text{SAT}_H = \left\{ \psi 01^n^{H(n)} : \psi \in \text{SAT}, \text{ and } n = |\psi| \right\},
\]

where H(n) is the smallest number i < log log n such that for every x ∈ {0, 1}^* with |x| ≤ log n, M_i outputs SAT_H within i|x|^1 steps. If there is no such number i then H(n) = log log n. (M_i denotes the i-th Turing machine.)

• (10 pts) Prove that the function H defined is computable in polynomial time.

• (15 pts) Prove that if SAT_H is NP-complete, then SAT is in P.

(Hint: Arora-Barak page 532. There is a scanned version posted in Piazza.)

Problem 4 [20 pts]. Show that 2SAT is in NL. (Hint: reduce 2SAT to PATH.)