Due: September 29th, 2017
Date: September 25, 2017

Assignment 2

You must submit it electronically to ELMS. This is a group assignment. Every group only needs to submit one solution. Group members get the same credit for the group submission.

This assignment is 7% in your total points. For the simplicity of the grading, the total points for the assignment is 70.

Problem 1 [10 pts]. Prove that if P=PSPACE, then EXP-TIME=EXP-SPACE. (Hint: use the padding technique.)

Problem 2 [15 pts]. Show that the following language is undecidable:

$$\left\{< M >: M \text{ is a machine that runs in } 100n^2 + 200, \text{ where } n \text{ is the input size}\right\}.$$

(Hint: use the diagonalization method or a reduction from the HALT problem.)

Problem 3 [25 pts]. Recall that the language SAT_H is defined by

$$\mathrm{SAT}_H = \left\{\psi 01^{n^{H(n)}} : \psi \in \mathrm{SAT}, \text{ and }, n = |\psi|\right\},$$

where $H(n)$ is the smallest number $i < \log\log n$ such that for every $x \in \{0,1\}^*$ with $|x| \leq \log n$, M_i outputs SAT_H within $i|x|^i$ steps. If there is no such number i then $H(n) = \log\log n$. (M_i denotes the ith Turing machine.)

- (10 pts) Prove that the function H defined is computable in polynomial time.
- (15 pts) Prove that if SAT_H is NP-complete, then SAT is in P.

(Hint: Arora-Barak page 532. There is a scanned version posted in Piazza.)

Problem 4 [20 pts]. Show that 2SAT is in NL. (Hint: reduce 2SAT to $\overline{\mathrm{PATH}}$.)
