Assignment 1

You must submit it electronically to ELMS. This is a group assignment. Every group only needs to submit one solution. Group members get the same credit for the group submission.

This assignment is 7% in your total points. For the simplicity of the grading, the total points for the assignment is 70.

Problem 1 [10 pts]. Let \( T(\cdot) \) be an increasing computable function. Show that there exists a decidable language \( A \) such that the following occurs:

If \( M \) is a TM that, on all inputs of length \( n \), halts in \( \leq T(n) \) time, then there is an INFINITE NUMBER of \( x \) such that \( A(x) \neq M(x) \). (Here we let \( A(x) = 1 \) if \( x \in A \) otherwise \( A(x) = 0 \).)

Problem 2 [10 pts]. Assume \( L_1, L_2 \in \text{NP} \) and \( S_1 \in \text{CoNP} \). Assume \( \text{NP} \neq \text{CoNP} \). Answer each of the following with proof or a counter-example.

(a) Is \( L_1 \cup L_2 \) necessarily in \( \text{NP} \)?
(b) Is \( L_1 \cup S_1 \) necessarily in \( \text{CoNP} \)?

Problem 3 [10 pts]. Assume that the decision problem of Graph Isomorphism is in \( \text{P} \). Show that the following function can be computed in poly-time. Given two graphs \( G_1, G_2 \),

- If \( G_1 \) and \( G_2 \) are not isomorphic than output NO.
- If \( G_1 \) and \( G_2 \) are isomorphic than output an isomorphism.

Problem 4 [20 pts]. Prove that the following language

\[
L = \{ (M, x, t) : \exists w \in \{0,1\}^t, s.t., M(x, w) \text{ halts within } t \text{ steps with output } 1 \}
\]

(where \( M \) is a deterministic Turing machine) is \( \text{NP} \)-complete.

Problem 5 [20 pts]. Prove that the following language is \( \text{NP} \)-complete

\[
3\text{COL} = \{ G : \text{there exists a coloring of } G \text{ with } 3 \text{ colors s.t. no adjacent vertices share the same color.} \}
\]

(Hint: You can assume 3SAT is \( \text{NP} \)-complete. You may use other resources but let me know what they are and you should hand in your own solution.)