Problem Set 9

Quantum Error Correction, 2018 spring
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Problem 1. Price of a stabilizer code

The price p of a stabilizer code is the volume of the smallest subsystem of qubits which supports all the logical operators.

(a) Find the price of the toric code defined with $N = 2L^2$ qubits.

(b) Find the price of the 15-qubit code.

(c) Prove that $p \leq n - d + 1$ and $p \geq k + d - 1$. (Hint: use the duality relation for the first inequality, and use the argument for proving the quantum Singleton bound for the second inequality).

Problem 2. Bound on local classical codes

Consider a classical stabilizer code in D dimensions. Show that

$$kd^{\frac{1}{D-1}} \leq O(n). \tag{1}$$

Here stabilizer generators are tensor products of Pauli-Z operators, and the classical code distance d is the smallest subsystem of "qubits" which supports all the X-type logical operators.

Problem 3. Symmetry in a stabilizer code

Consider a stabilizer code with $k = 1$ defined on a D-dimensional hypercubic lattice ($N = L^D$). Assume that the stabilizer group $\mathcal{S}$ is invariant under finite translations:

$$T_1^{c_1}(\mathcal{S}) = \cdots = T_D^{c_D}(\mathcal{S}) = \mathcal{S} \tag{2}$$

where T_j are operators that shift qubits in the direction of $\hat{j}$. Here c_j are $O(1)$ constants. Let d_X, d_Z be the sizes of the smallest subsystem of qubits which support a logical-X and logical-Z operators respectively. Show that

$$d_X d_Z \geq O(N). \tag{3}$$

(Hint: We do not need to assume locality of stabilizer generators in this problem).