Problem Set #2

Quantum Error Correction
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Due Thursday, Jan. 18, 2018

Problem #1. Example stabilizer

For each of the following sets of Paulis, determine if they define valid stabilizers. If so, give their parameters $[[n, k, d]]$.

a) Stabilizer is all products of these operators:

$$\begin{matrix} X & X & Z & Y & I \\ Z & Y & I & I & X \\ X & I & X & Z & Z \end{matrix}$$

b) Stabilizer is all products of these operators:

$$\begin{matrix} X & X & X & X & X & X \\ Y & Y & Y & Y & Y & Y \\ Z & Z & Z & Z & Z & Z \end{matrix}$$

c) In binary symplectic matrix form:

$$\left(\begin{array}{cccccc|cccccc} 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \end{array}\right)$$

d) The stabilizer corresponding to the GF(4) linear code with the following parity check matrix:

$$\begin{pmatrix} 0 & 1 & 1 & \omega & \omega^2 \end{pmatrix}$$

Problem #2. Stabilizer generating sets

Suppose we have a set of stabilizer generators $\{M_1, \ldots, M_r\}$ for a stabilizer S and $N \in S$ is not a generator. Show that we can remove an element of the original generating set and replace it with N to get a new minimal generating set.

Problem #3. Low-density parity check CSS codes

A classical LDPC ("low density parity check") code is an $[n, k, d]$ linear code where each row of the parity check matrix has at most r 1's and each column of the parity check matrix has at most c 1's, with r and c of constant size (as n gets large). (Sometimes LDPC codes with r and c increasing sublinearly with n are also considered, but assume r and c are constant for the purposes of this problem.) Classical LDPC codes are interesting because they can achieve good values of k/n, d/n, and also generally have good decoding algorithms.

A quantum LDPC code is a stabilizer code for which each generator has low weight and each qubit appears in only a small number of generators. One might try to make good quantum LDPC codes using the CSS construction, based on pairs of classical LDPC codes $C_1(n)$ and $C_2(n)$. Suppose that one finds a family of such codes which produce $[[n, k, d]]$ quantum codes with k/n and d/n both constant as n gets large. Show that this family of quantum codes must be degenerate for large n.

[No such family is known in the quantum case. The point of the problem is that, because degeneracy is important to find such codes, the quantum case is not a straightforward application of the CSS construction.]