Computergrafik

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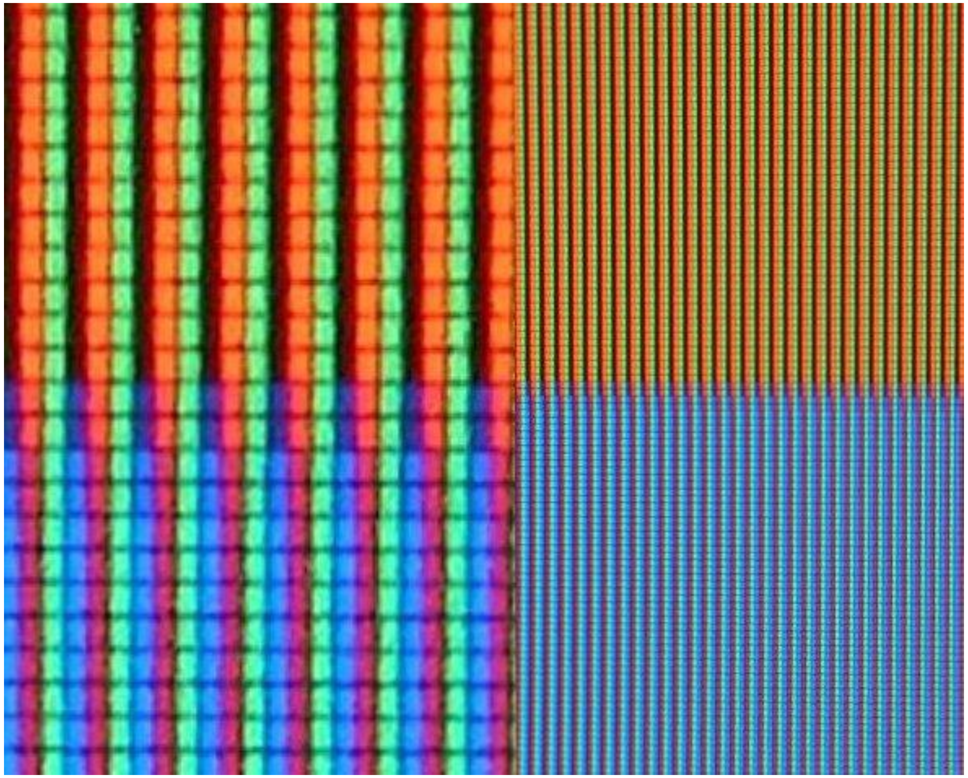
Universität Bern
Herbst 2016

Today

Color

- What is the relation between RGB values sent to a monitor and the colors we see?



RGB subpixels of LCD displays

Today

Color

- Physics background
- Color perception
- Color spaces
- Color reproduction on monitors
- Perceptually uniform color spaces

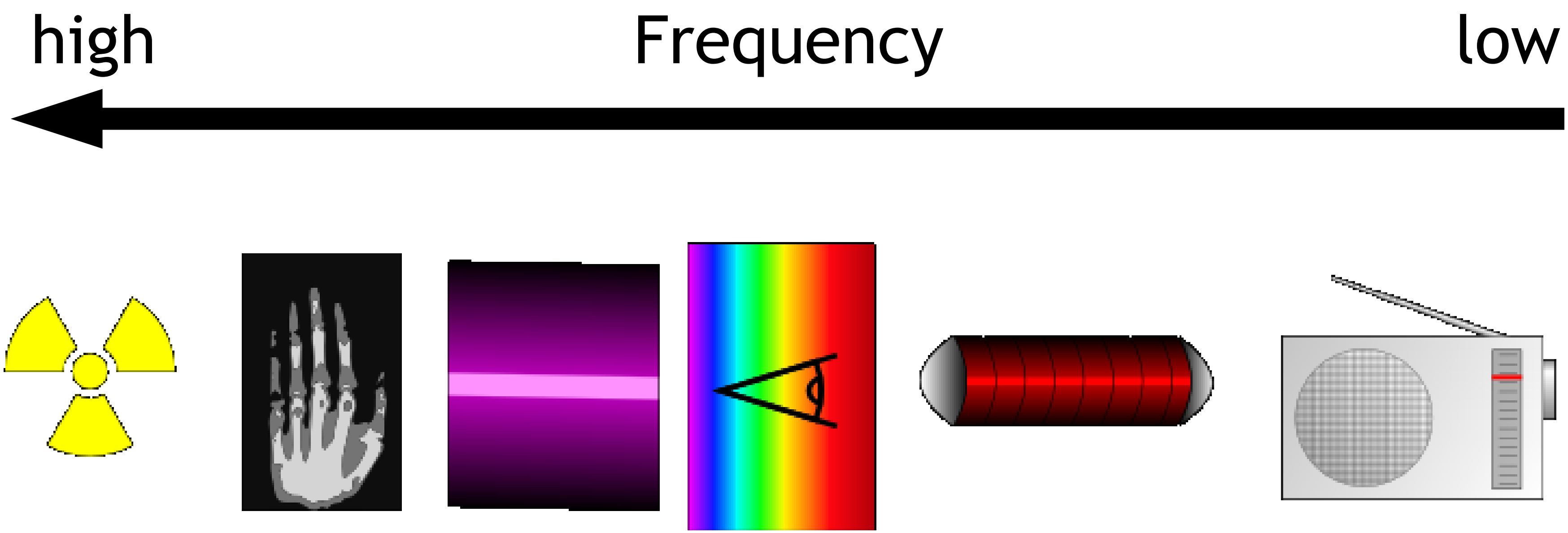
Light: physical models

- Corpuscular theory (particles) [Newton ~1700]
- Wave theory [Huygens ~1700; Young 1801]
- Electromagnetic waves [Maxwell 1862]
- Photons (tiny particles) [Planck 1900]
- Wave-particle duality [Einstein, early 1900]
  "It depends on the experiment you are doing whether light behaves as particles or waves"
- Simplified models in computer graphics

Electromagnetic waves

- Different frequencies

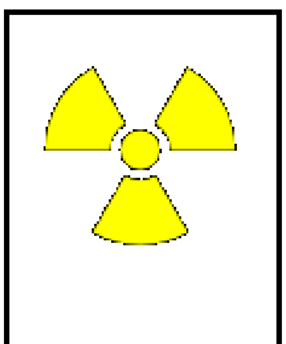
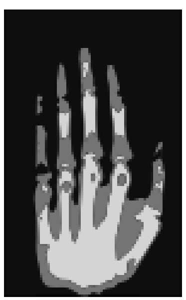
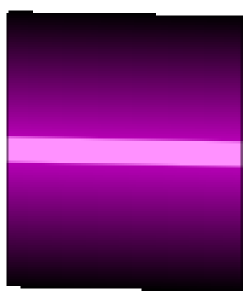
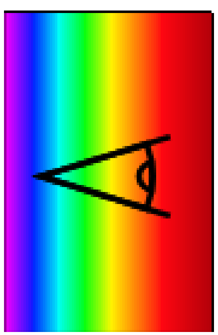
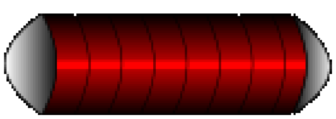
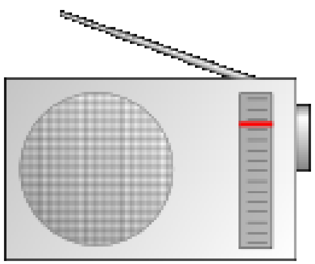
  http://en.wikipedia.org/wiki/Electromagnetic_radiation
  http://en.wikipedia.org/wiki/Electromagnetic_spectrum



Electromagnetic waves

- Different frequencies
  http://en.wikipedia.org/wiki/Electromagnetic_radiation
  http://en.wikipedia.org/wiki/Electromagnetic_spectrum

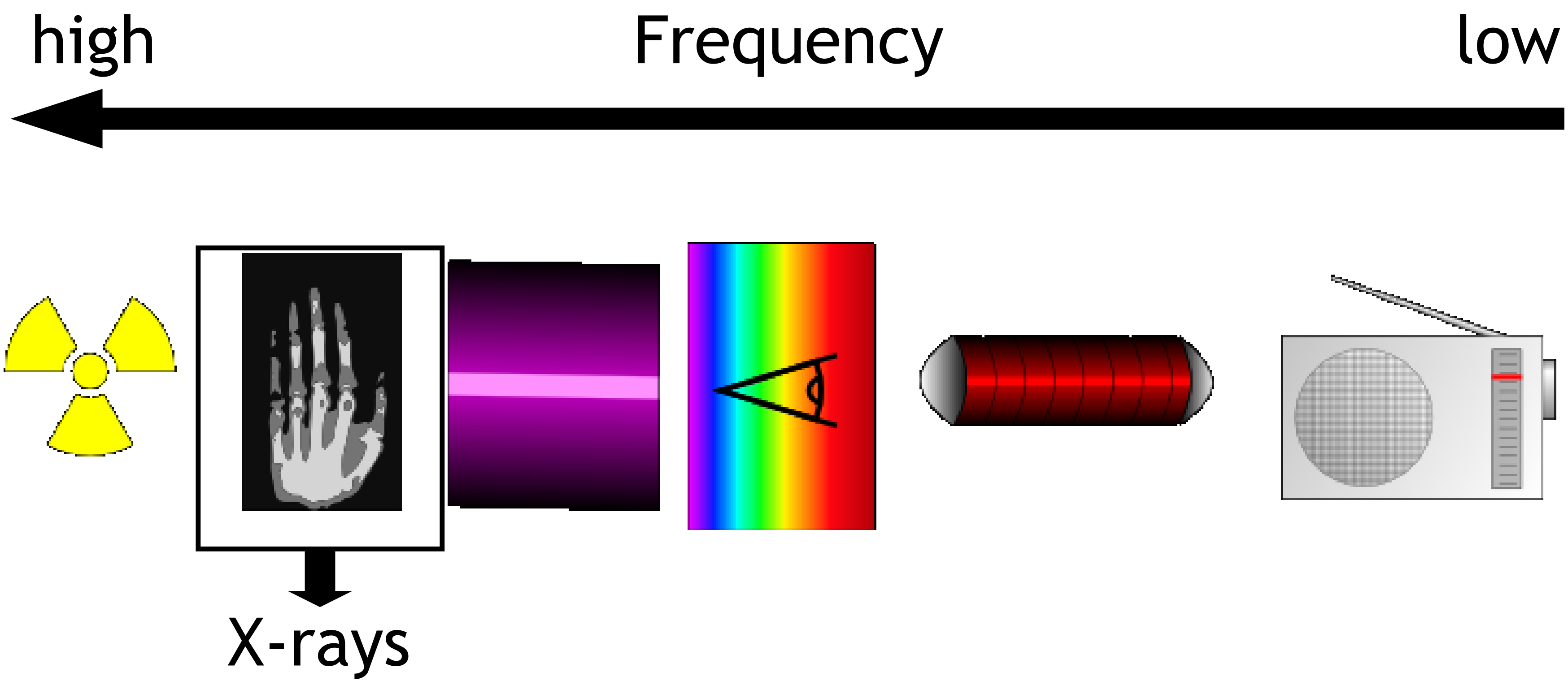
high Frequency low

Gamma rays,
nuclear radiation

Electromagnetic waves

- Different frequencies
  http://en.wikipedia.org/wiki/Electromagnetic_radiation
  http://en.wikipedia.org/wiki/Electromagnetic_spectrum



Electromagnetic waves

- Different frequencies
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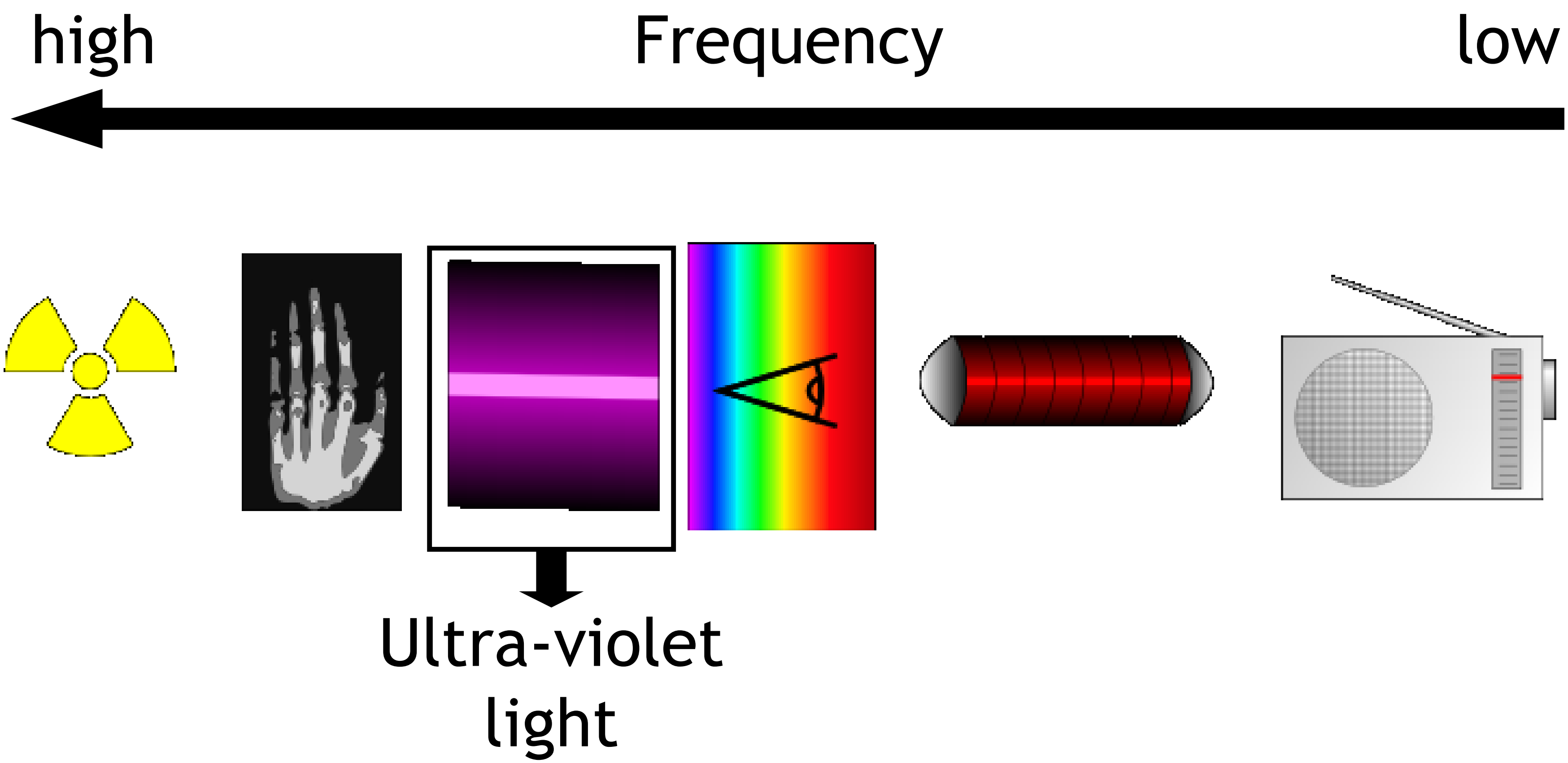

Electromagnetic waves

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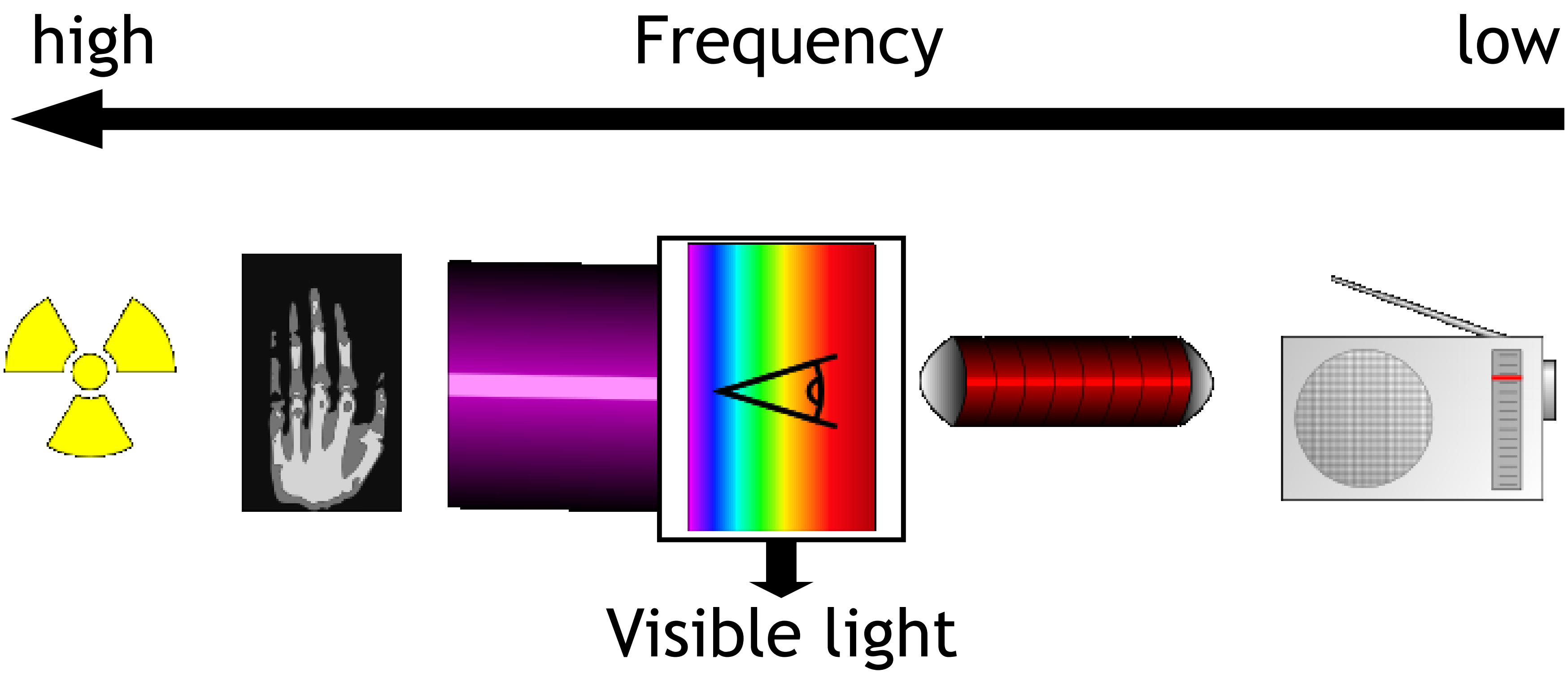

Electromagnetic waves

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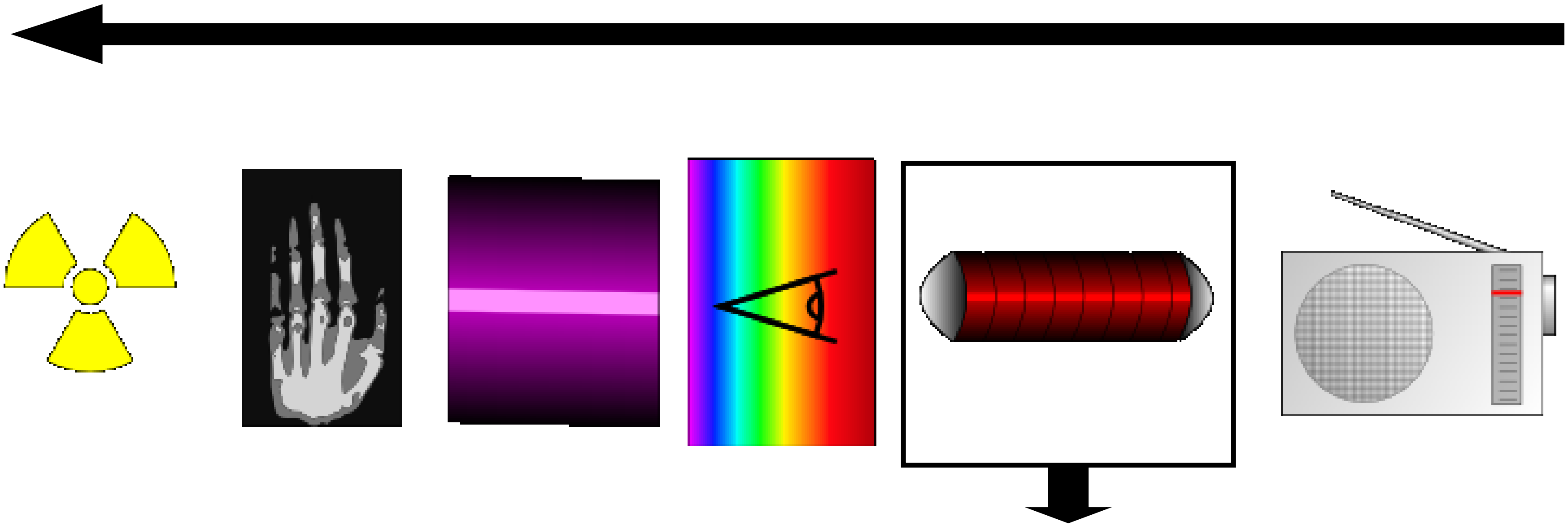
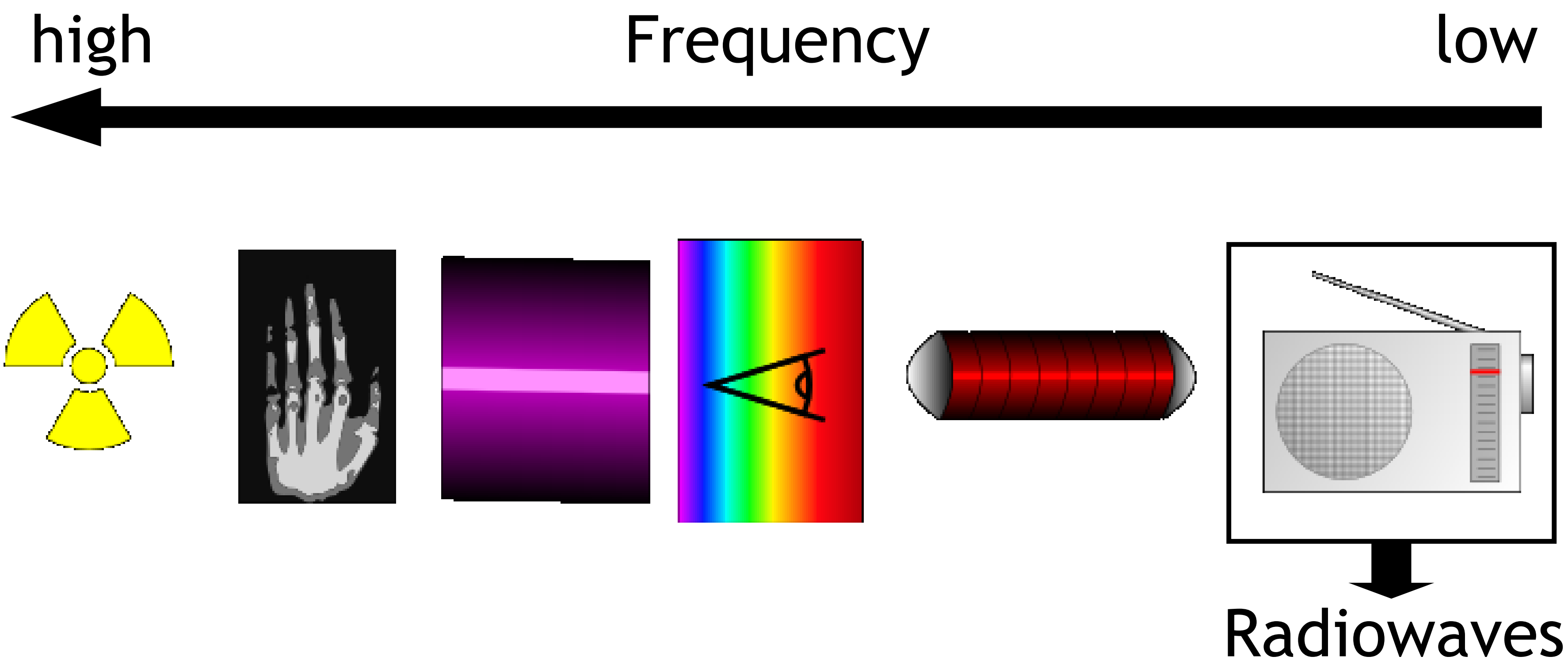


Infrared,
microwaves

Electromagnetic waves

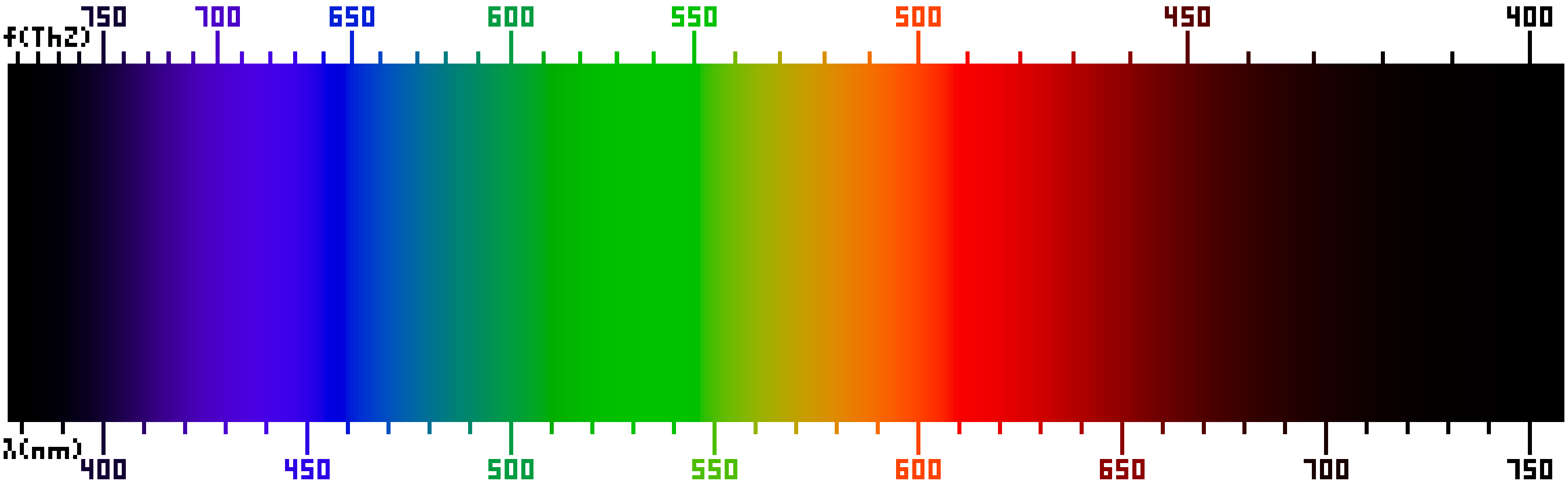
- Different frequencies

  http://en.wikipedia.org/wiki/Electromagnetic_radiation
  http://en.wikipedia.org/wiki/Electromagnetic_spectrum



Visible light

Frequency



Wavelength

1nm=10^-9 meters,

one-billionth of a meter

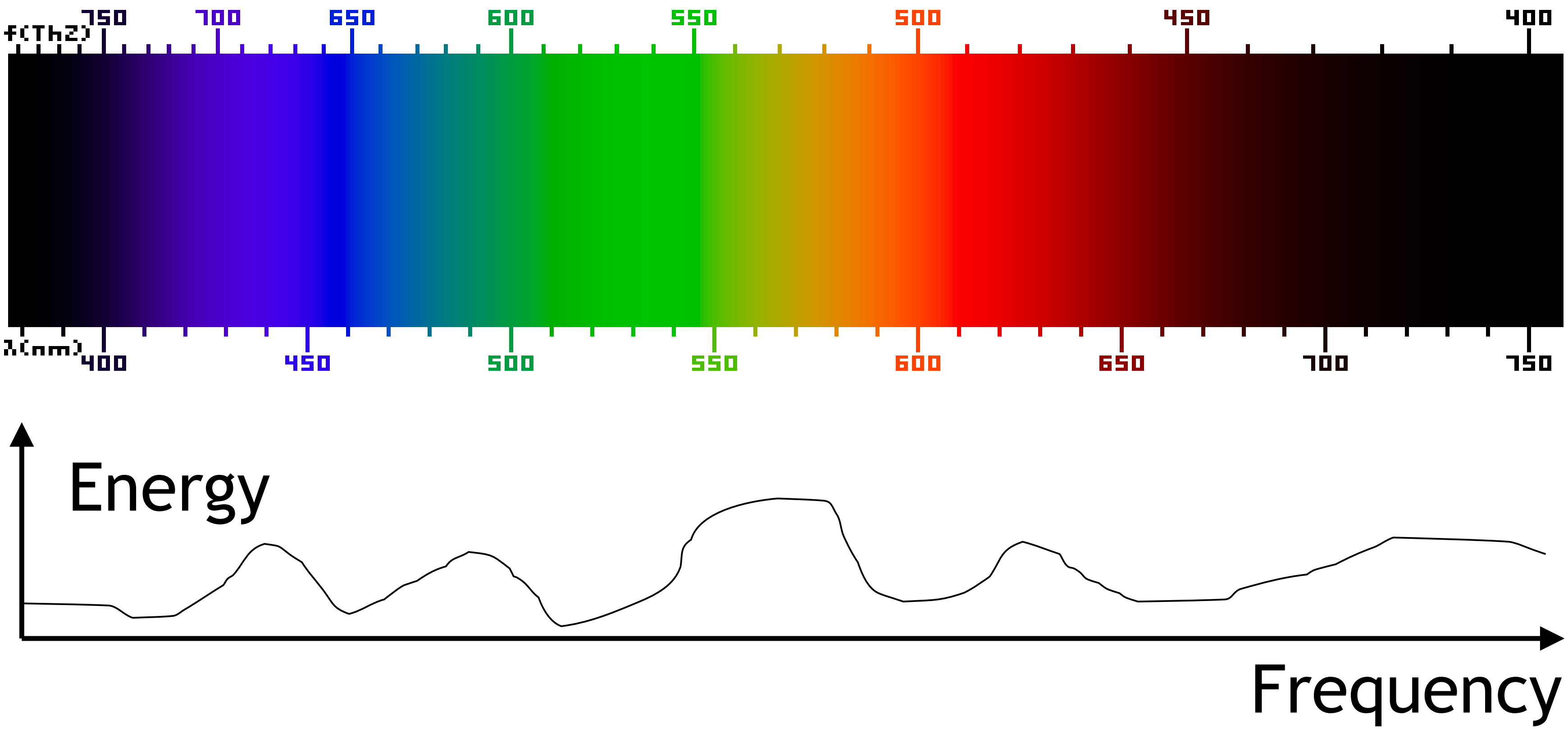
speed of light = wavelength * frequency

Light transport: Geometrical optics

- Simplified model
  http://en.wikipedia.org/wiki/Geometrical_optics
- Roughly speaking
  - Light is transported along straight rays
  - When light interacts with material, it may be reflected or refracted
- Can model most light transport effects that are important for our daily experience

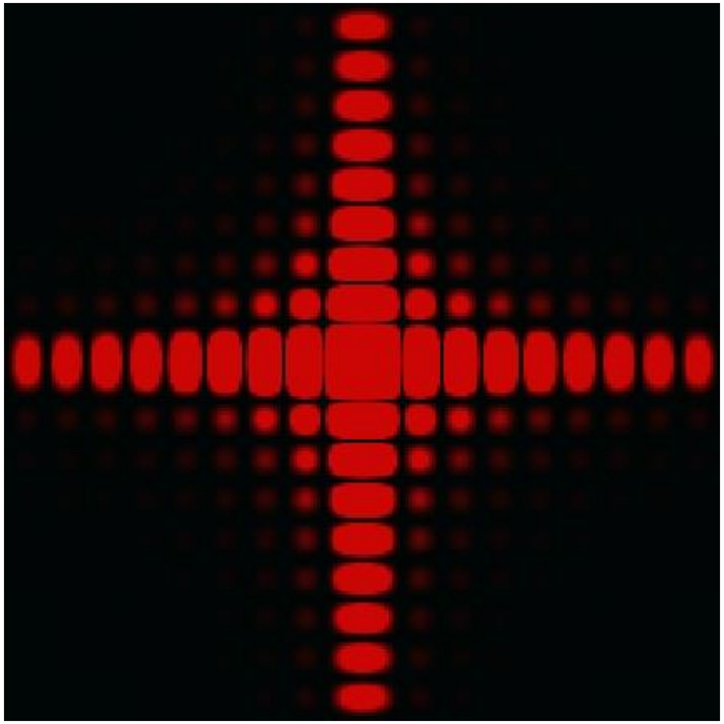
Light transport: Geometrical optics

- Rays carry a spectrum of electromagnetic energy
  - An “energy spectrum”, energy per wavelength



Limitations

- „Wave nature" of light ignored
- E.g., no diffraction effects



Diffraction pattern of a small square aperture

Surface of a DVD forms a diffraction grating

Today

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- Color perception
- Color spaces
- Color reproduction on monitors
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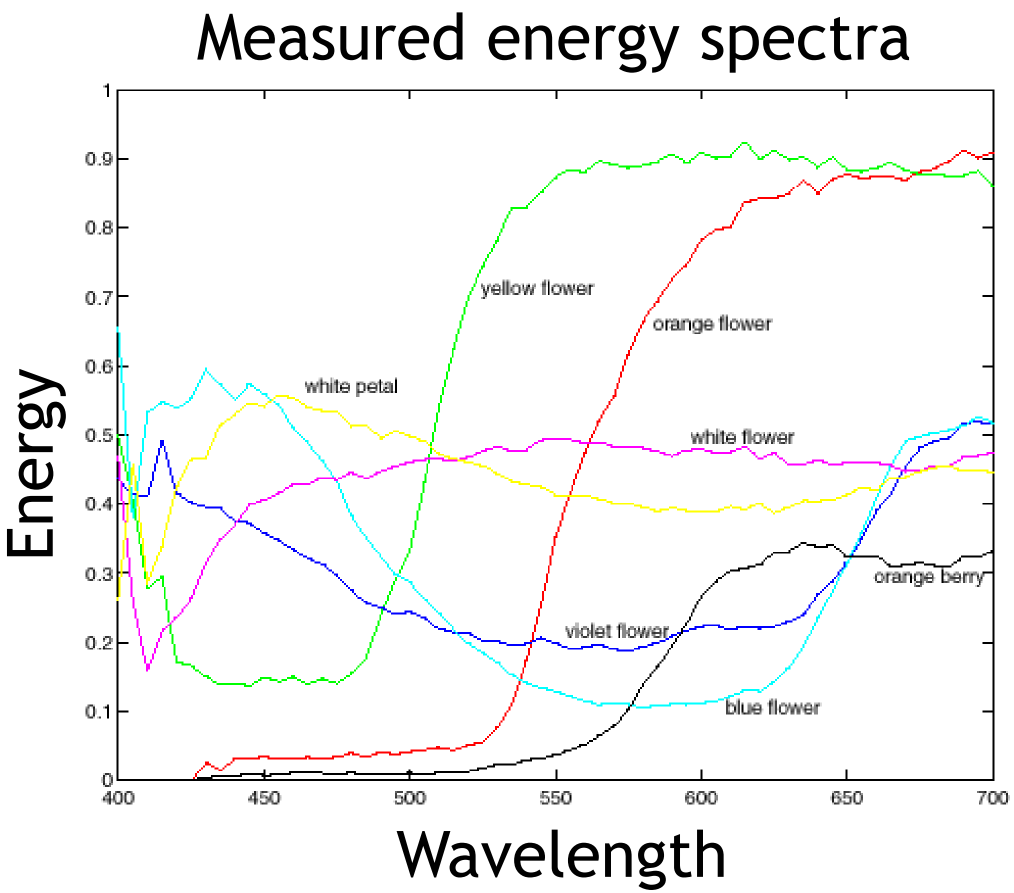
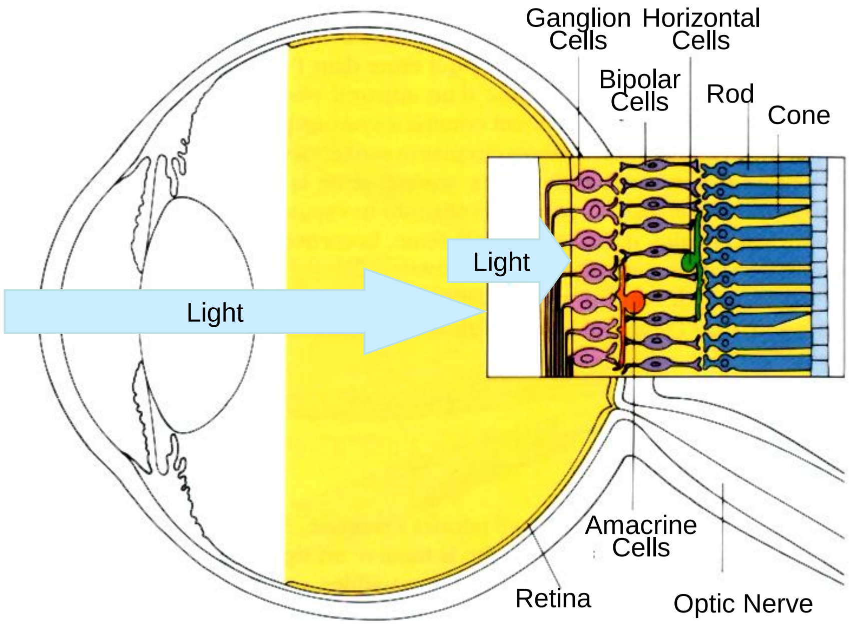
Light and color



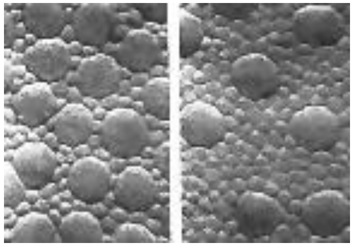
- How do humans perceive/measure such energy spectra?
- Can we distinguish any two different spectra?

Color perception

- Photoreceptor cells
  http://en.wikipedia.org/wiki/Photoreceptor_cell
- Light sensitive
- Two types, rods and cones




Rods Cones


Distribution of Cones and Rods

Photoreceptor cells

Rods

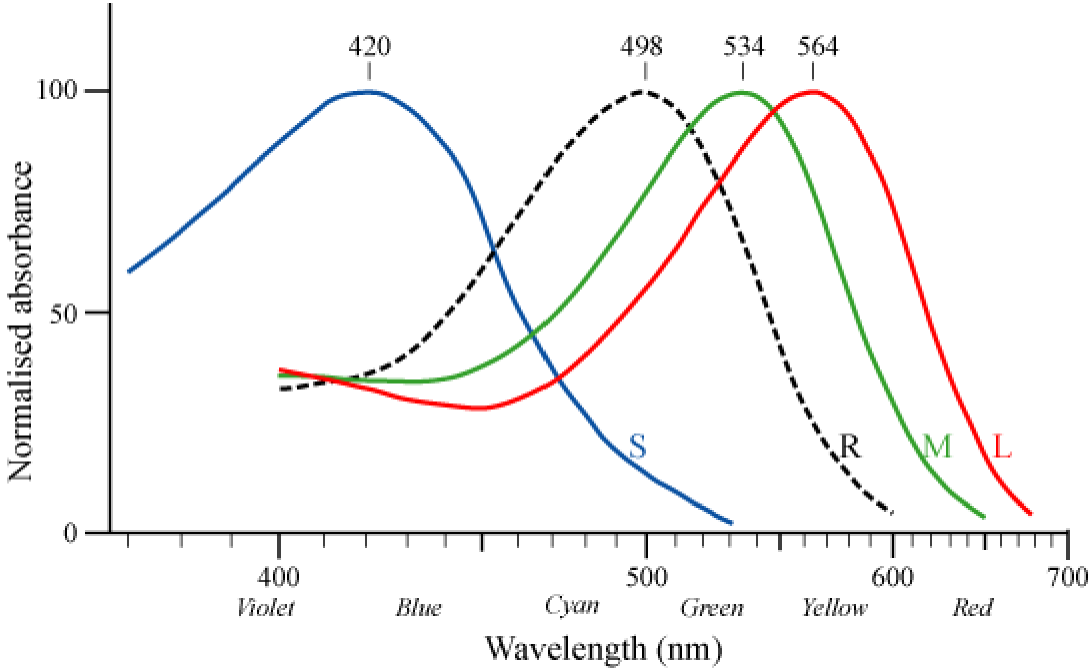
- About 100 times more sensitive than cones
- Low light vision
- Not involved in color perception

Cones

- 3 types of cones: S,M,L
- Sensitive to different wavelengths (short, medium, long)

Photoreceptor cells

- Absorbance: fraction of light absorbed at each wavelength
- Absorbance curves S, M, L for SML cone pigments



http://en.wikipedia.org/wiki/Photoreceptor_cell

- Experimentally determined in the 1980s
  http://www.pubmedcentral.nih.gov/articlerender.fcgi?artid=1279132

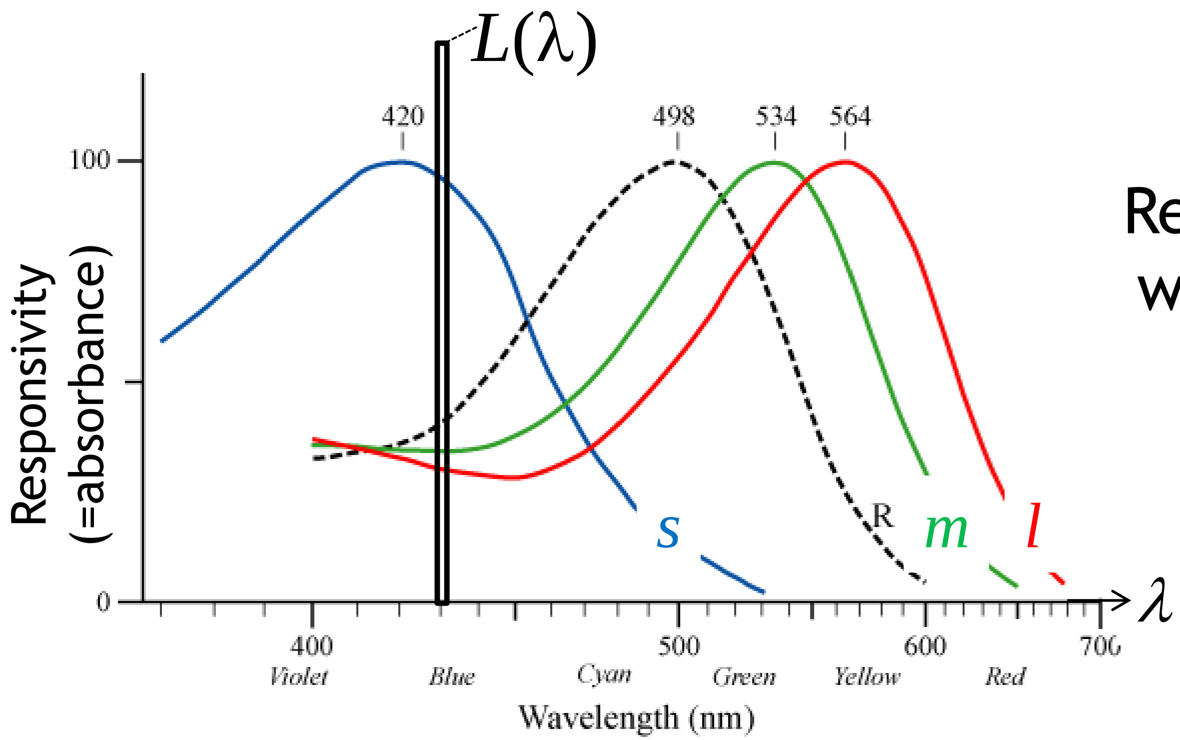
Response to arbitrary spectrum

- Response
  - Neural signal emitted by cell upon stimulus
  - "How strongly the cell reacts to a stimulus"
- Simplified model for response of cone receptors to given spectrum (stimulus), derived using two assumptions

1. SML absorbance at each wavelength = responsivity (input-output gain) at that wavelength
   http://en.wikipedia.org/wiki/Responsivity
2. Grassmann's law: linearity
   http://en.wikipedia.org/wiki/Grassmann's_law_(optics)

1. Responsivity = absorbance

- Assume stimulus $L(\lambda)$ is spectral color („peak“) at wavelength λ, http://en.wikipedia.org/wiki/Spectral_color



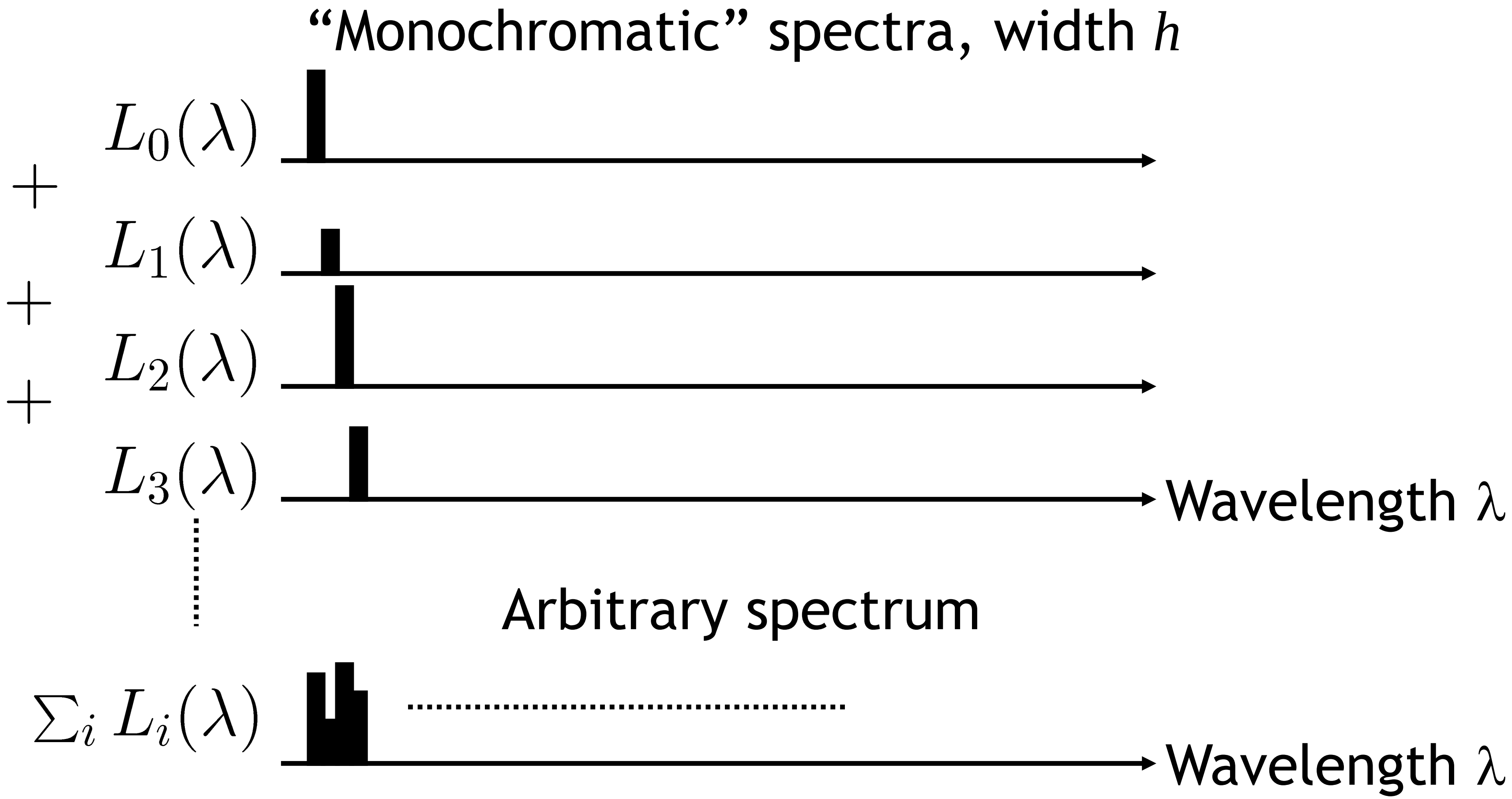
Responsivity at each wavelength defines response curves $s(\lambda)$, $m(\lambda)$, $l(\lambda)$

- SML responses

$$\text{response}_s\left(L(\lambda)\right) = s(\lambda)L(\lambda)$$
$$\text{response}_m\left(L(\lambda)\right) = m(\lambda)L(\lambda)$$
$$\text{response}_l\left(L(\lambda)\right) = l(\lambda)L(\lambda)$$

2. Linearity

- Arbitrary spectrum as sum of "mono-chromatic peaks"



2. Linearity

- Grassman's law (linearity): Response to sum of spectra is equal to sum of responses to each spectrum

$$\text{response}\left(\sum_i L_i(\lambda)\right) = \sum_i \text{response}\left(L_i(\lambda)\right)$$

- Remember: if $L_i(\lambda)$ is a monochromatic peak at wavelength λ_i, then

$$\text{response}_s\left(L_i(\lambda)\right) = s(\lambda_i)L_i(\lambda_i)$$

- In the limit (infinitely many peaks)

$$\text{response}_s\left(\sum_i^{\infty} L_i(\lambda_i)\right) = \sum_i^{\infty} s(\lambda_i)L_i(\lambda_i) = \int s(\lambda)L(\lambda)d\lambda$$

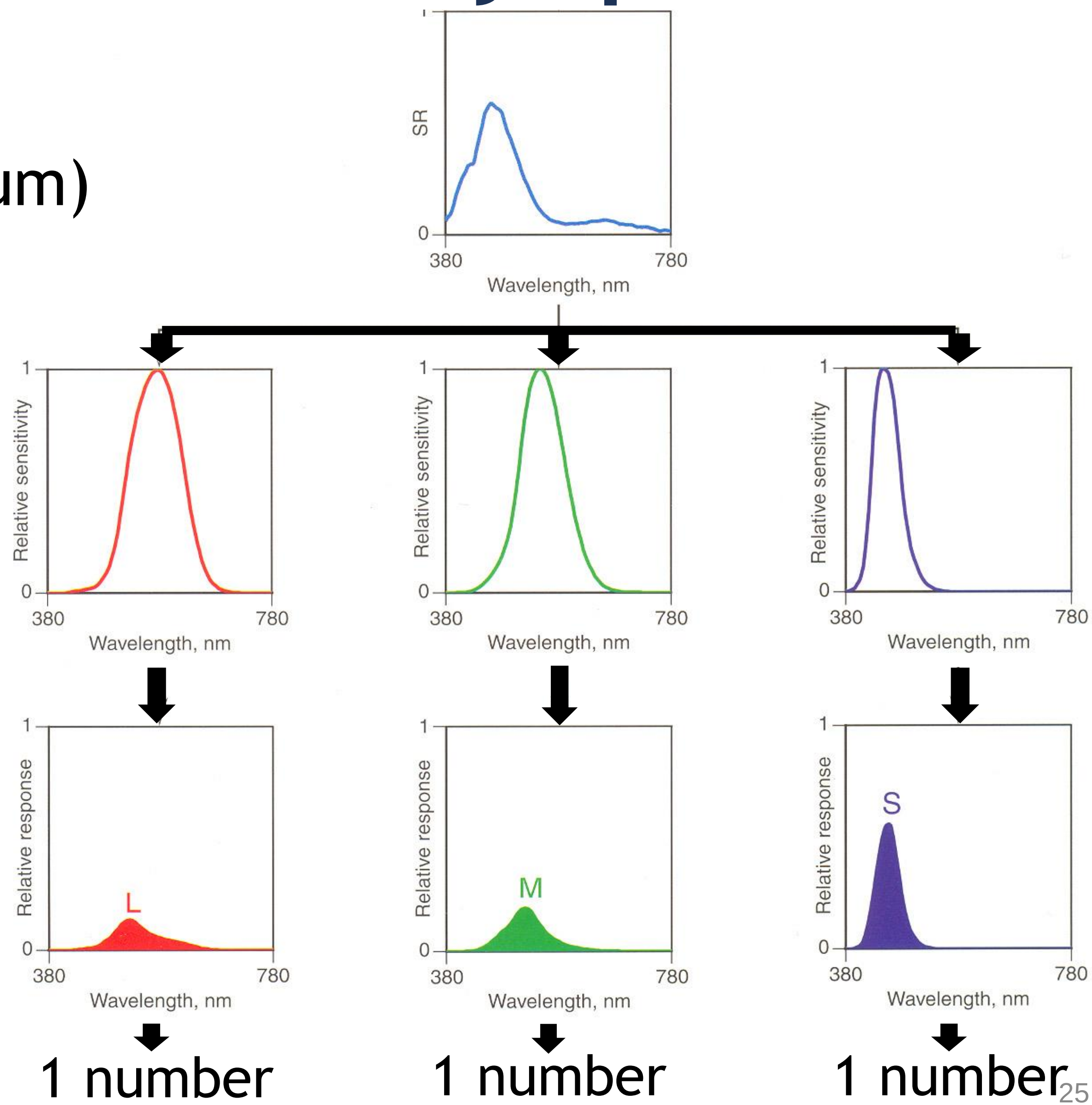
Response to arbitrary spectrum

Stimulus
(arbitrary spectrum)

Response curves

Multiply

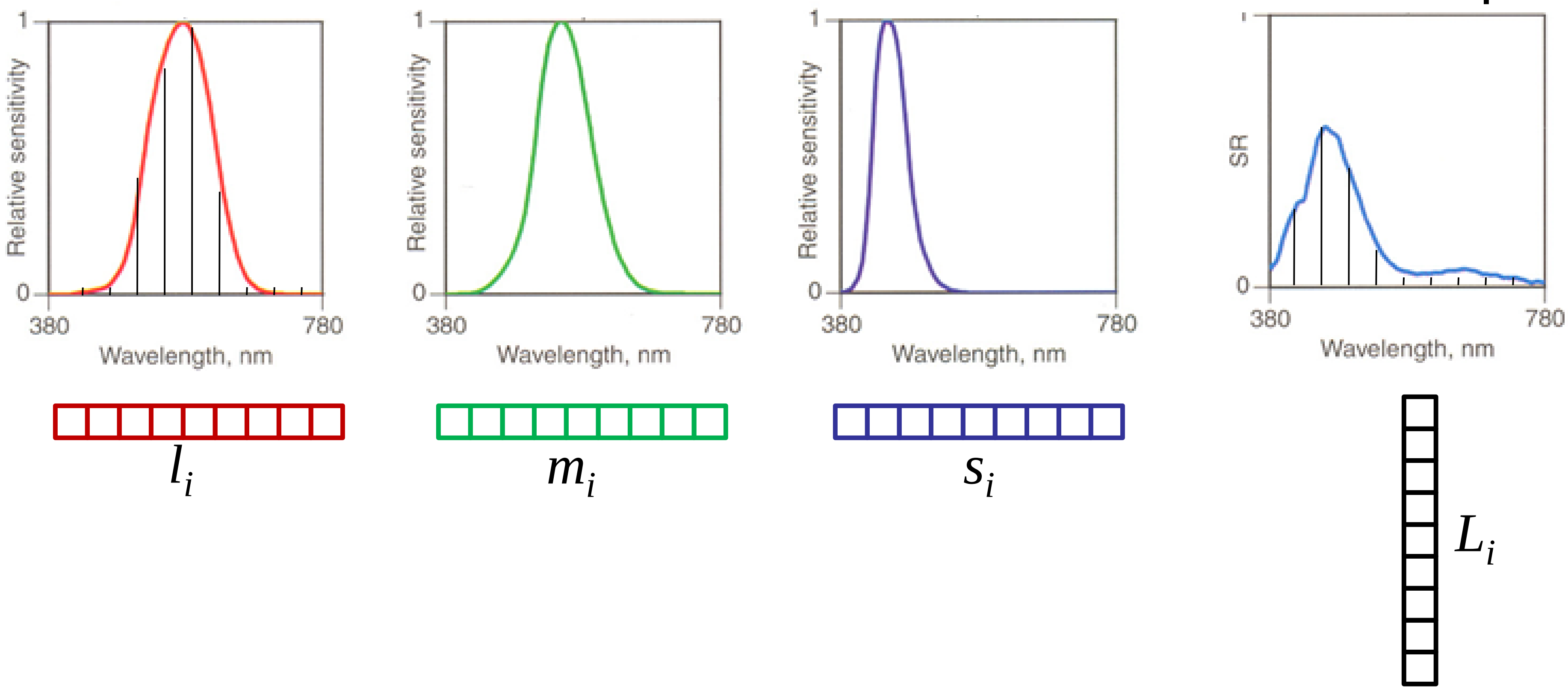
Integrate



Discrete case

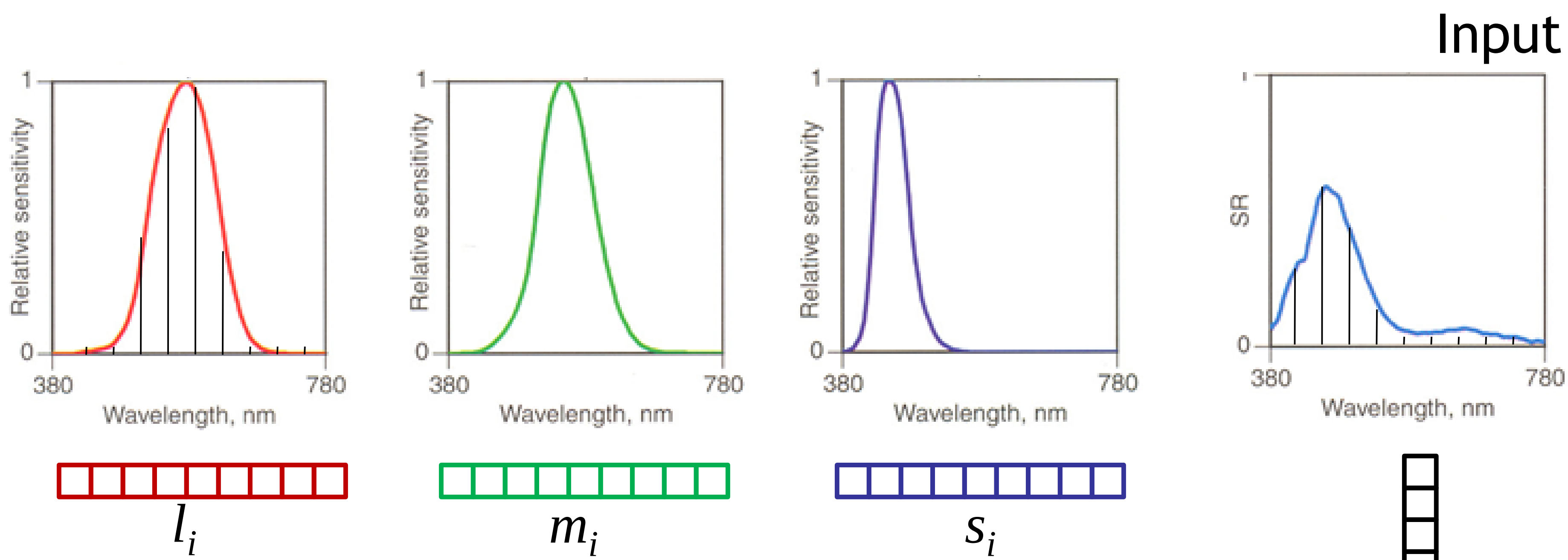
- Cones and input spectrum represented as vectors

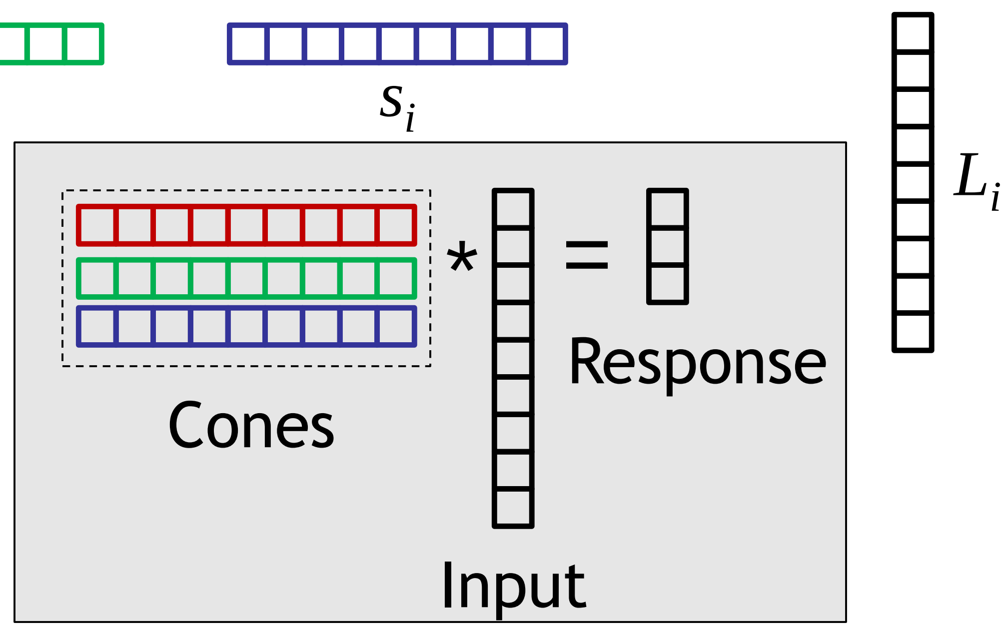
Input



Discrete case

- Cones and input spectrum represented as vectors

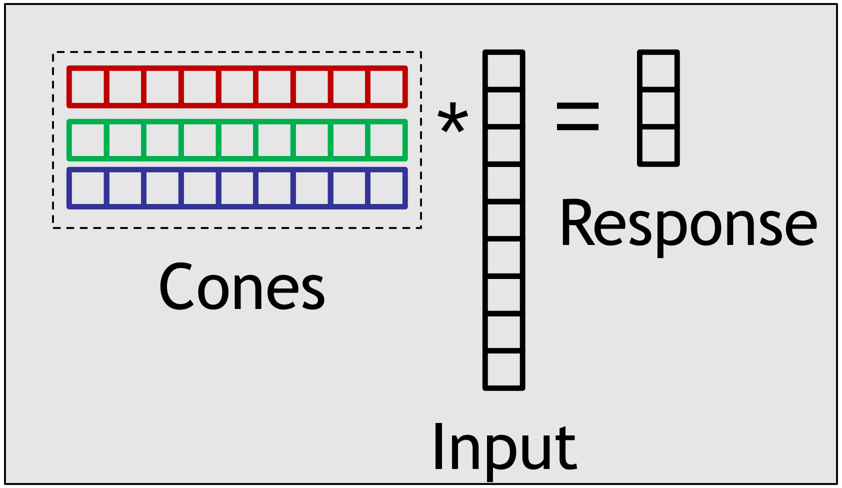




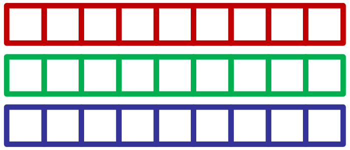
- Response is matrix-vector multiplication
- Integration is now summation

Discrete case

- Response is matrix-vector multiplication
- Linear map!



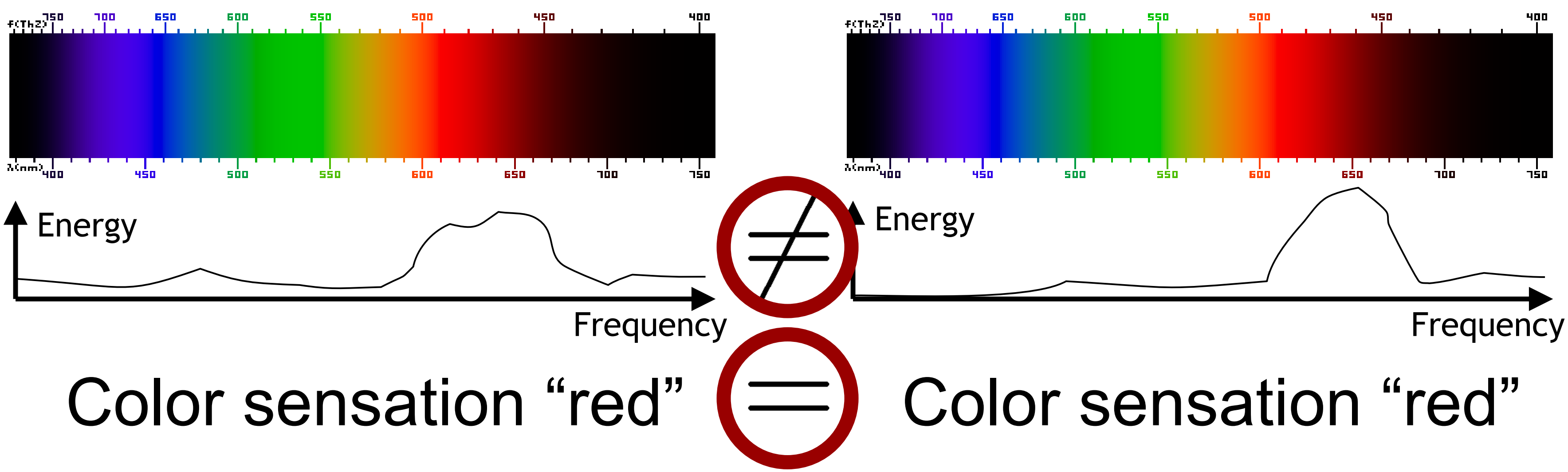
- Projection of input onto three basis vectors
  - Basis vectors in general not orthogonal



Metamerism

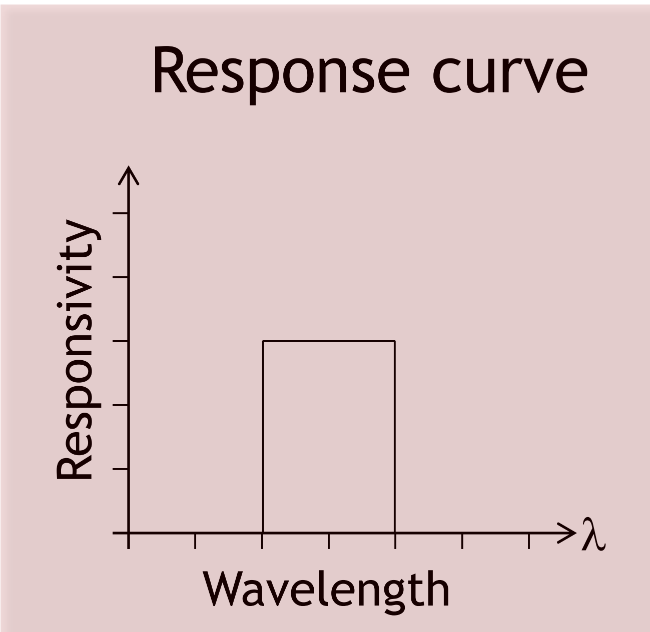
http://en.wikipedia.org/wiki/Metamerism_%28color%29

- Different spectra, same response
  - Such spectra are called metamers
- Intuitive reason for metamerism
  - Arbitrary spectrum is “infinite dimensional” (has infinite number of degrees of freedom)
  - Response has three dimensions
  - Loose information

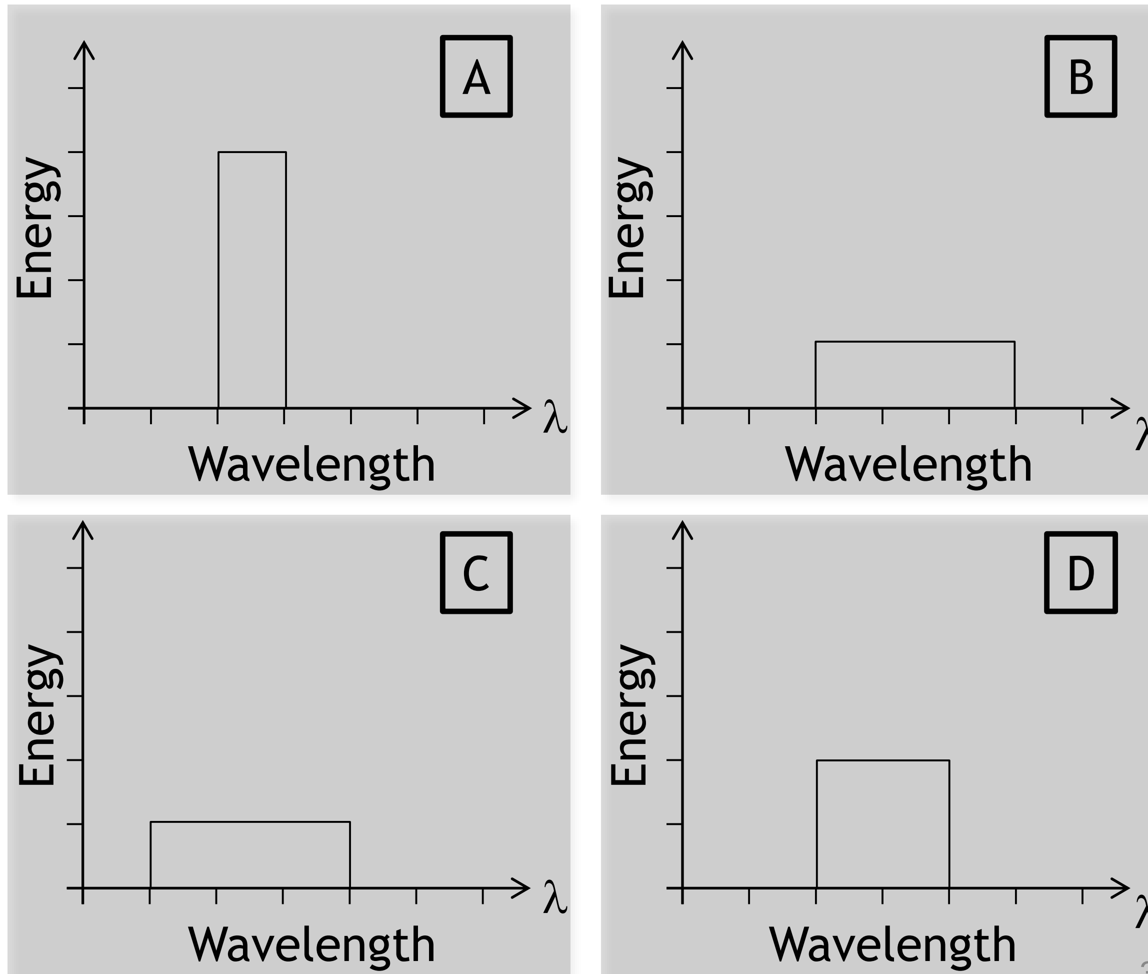


Metamers

- Which stimuli are metamers?

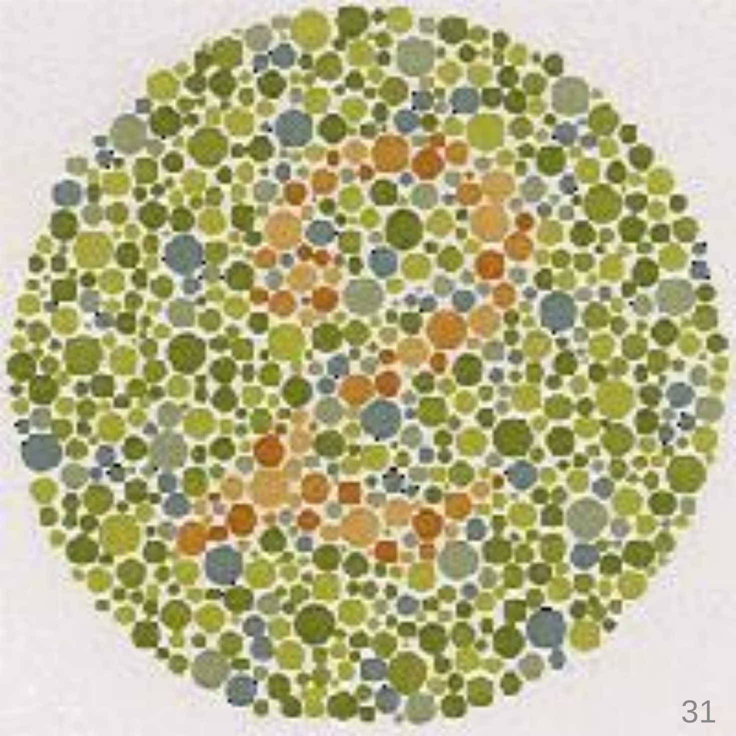


Stimuli



Color blindness

- One type of cone missing, damaged
- Different types of color blindness, depending on type of cone
- Can distinguish even fewer colors
- But we are all a little color blind…
- Simulate color blindness http://www.vischeck.com/



Note

- Color perception is much more complicated than LMS cones…
- LMS cone responses just input to further processing
- Visual pathway

LMS cone responses

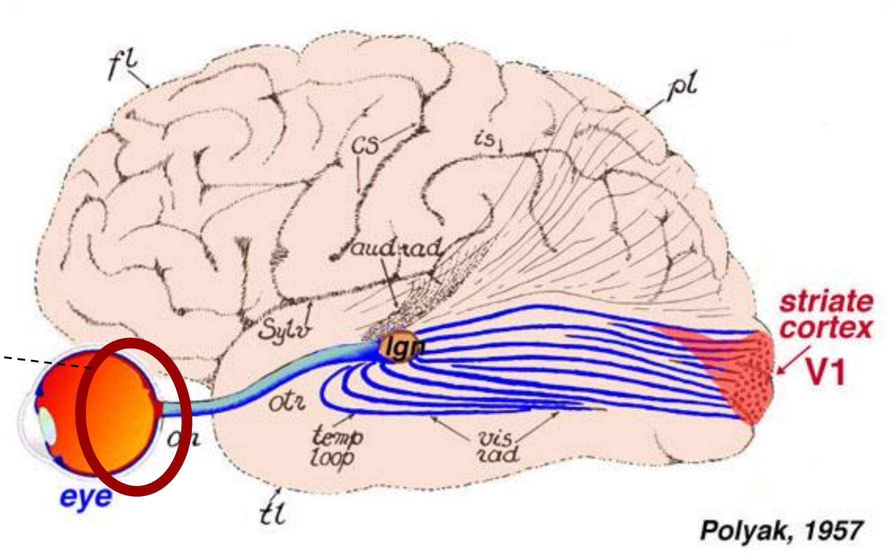


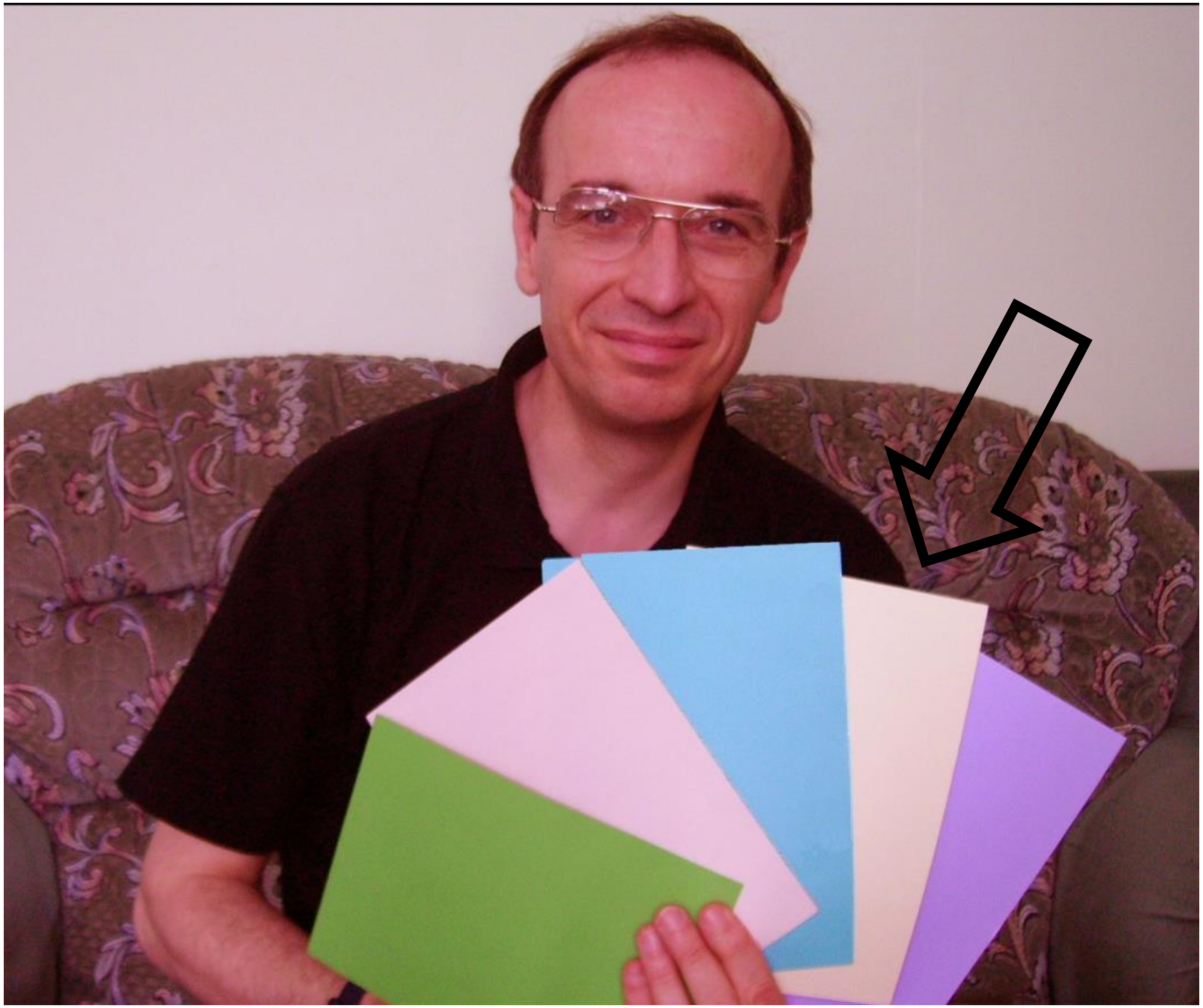
Figure 8. Visual input to the brain goes from eye to LGN and then to primary visual cortex, or area V1, which is located in the posterior of the occipital lobe. Adapted from Polyak (1957).

Opponent process theory

- After sensing by cones, colors are encoded as red versus green, blue versus yellow, and black versus white
  http://en.wikipedia.org/wiki/Opponent_process
- First proposed in 19^{th} century
- Physiological evidence found in the 1950s

Color constancy (chromatic adaptation)

- What color is this sheet of paper?



Color constancy (chromatic adaptation)

- Color of object is perceived as the same even under varying illumination
  - White object under red illumination reflects red light, yet it is still perceived as white

http://en.wikipedia.org/wiki/Color_constancy

Nonlinear processing

- Perceived contrast is related to ratio of brightness, not absolute difference
  - Two colors with (physical) brightness b1 and b2
  - Human visual system "computes" log(b1/b2) = log(b1)-log(b2), not b1-b2
- Many more complicated effects

http://www.johnsadowski.com/big_spanish_castle.html

http://www.johnsadowski.com/big_spanish_castle.html

Summary

- Simplified model for initial physiological response to light
  - Response curves based on absorbance of SML cone pigments
  - Grassmann‘s law: linearity
- But:
  „Color perception is not just the measurement of the spectrum of a single light ray, but depends on the spatial and temporal context of the scene“

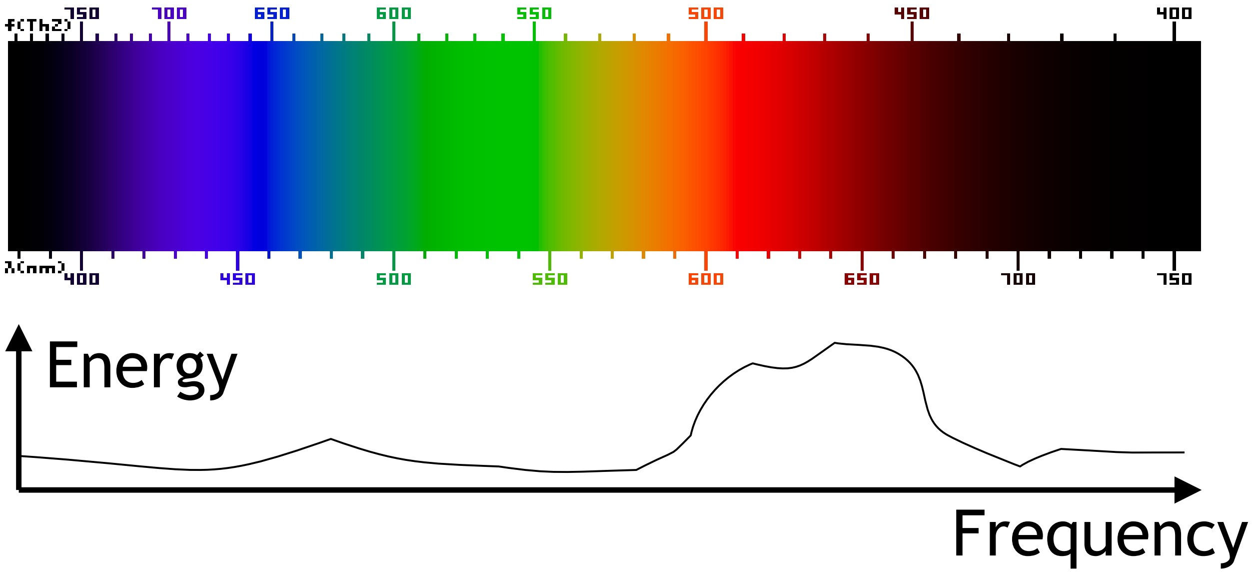
Today

Color

- Physics background
- Color perception
- Color spaces
- Color reproduction on monitors
- Perceptually uniform color spaces

Color spaces

- How can we represent, reproduce color?
- Brute force: store full spectral energy distribution
  - Disadvantages?



Color spaces

- Representation should be complete, but as compact as possible
  - Any pair of colors that can be distinguished by humans should have two different representations
  - Any pair of colors that appears the same to humans should have the same representation

Color spaces

“Putting numbers on colors”

- Set of parameters describing a color sensation
- “Coordinate system” for colors
- Three types of cones, expect three parameters to be sufficient
- Why not use L,M,S cone absorptions according to model from before?
  - Historical reasons, not known until 1980s

Trichromatic theory

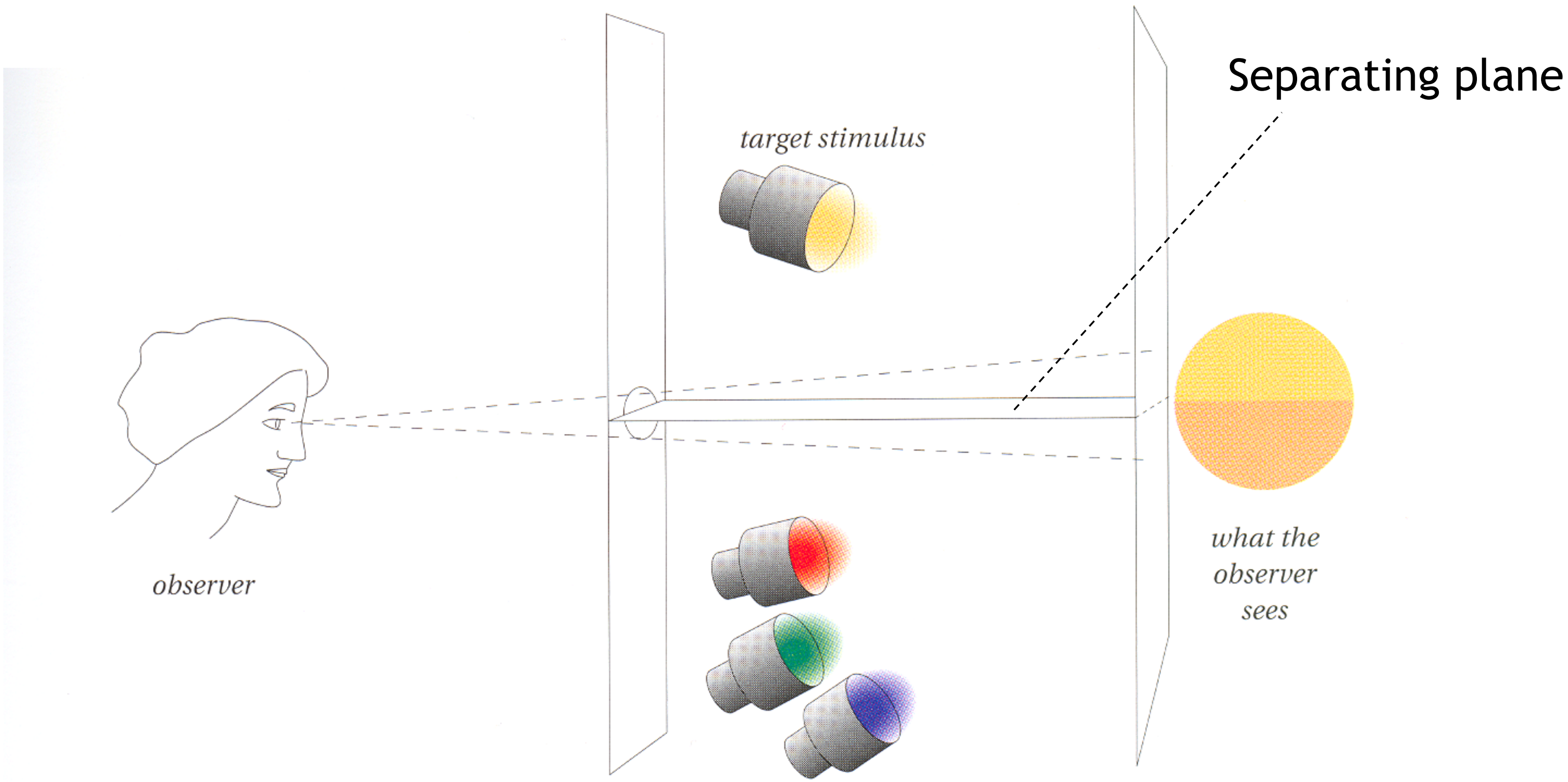
- Claims any color can be represented as a weighted sum of three primary colors
- Propose red, green, blue as primaries
- Developed in 18^{th}, 19^{th} century, before discovery of photoreceptor cells (Thomas Young, Hermann von Helmholtz)
- Not quite true, but good intuition for defining color spaces

Tristimulus experiment

- Given arbitrary color, want to know the weights for the three primaries to match the color
- Weights called tristimulus values
  - Use these as “color coordinates”
- Experiments by CIE, circa 1920
  - Commission Internationale de l’Eclairage, International Commission on Illumination
    http://en.wikipedia.org/wiki/CIE_1931_color_space

Tristimulus experiment

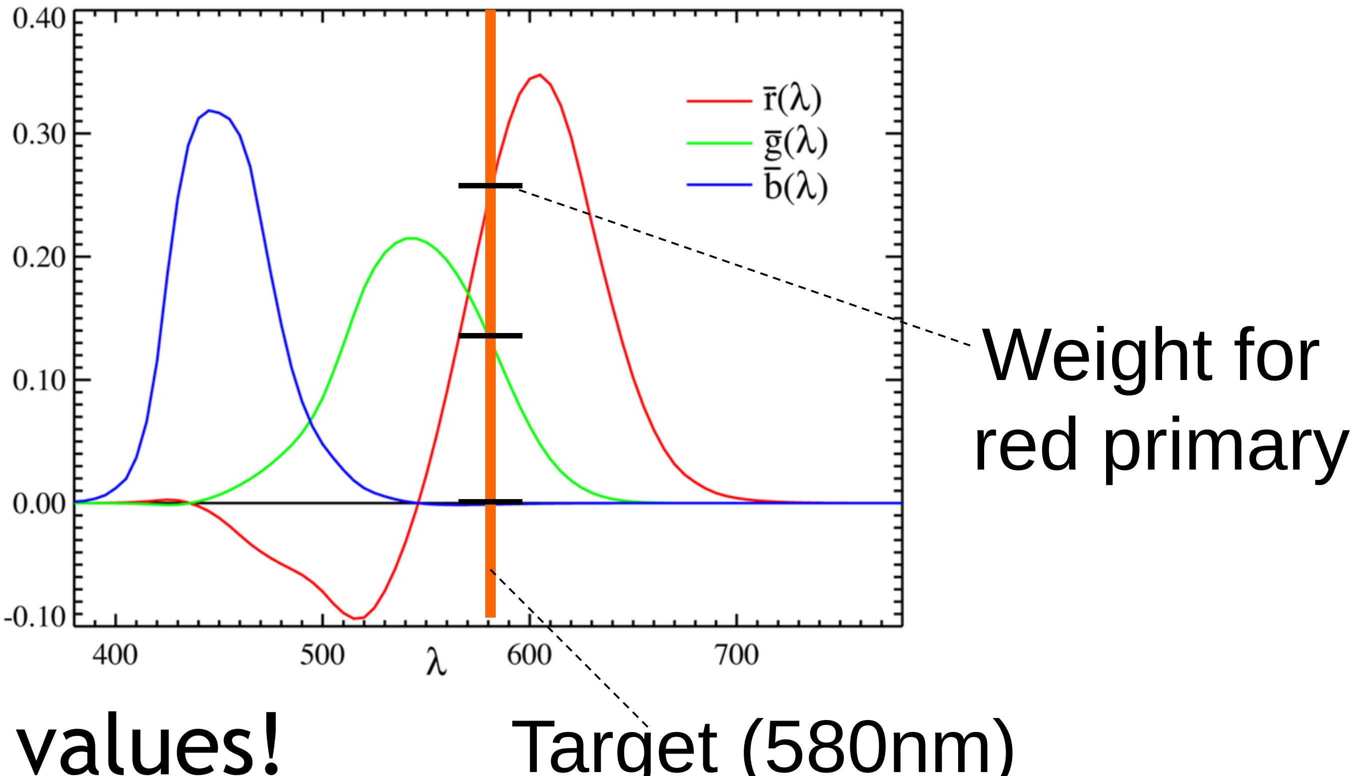
- Goal: find tristimulus values for spectral target colors
  http://en.wikipedia.org/wiki/Spectral_color
- Setup: well defined standard viewing conditions



The observer adjusts the intensities of the red, green, and blue lamps until they match the target stimulus on the split screen.

Tristimulus experiment

- Spectral primary colors were chosen
  - Blue (435.8nm), green (546.1nm), red (700nm)
- Matching curves for monochromatic target



- Negative values!

Tristimulus experiment

Negative values

- Some spectral colors could not be matched by primaries in the experiment
- "Trick"
  - One primary could be added to the source
  - Match with the other two
  - Weight of primary added to the source is considered negative

Photoreceptor absorption vs. matching curve

- Not the same!

Tristimulus values

- Goal: given arbitrary spectrum, find weights R, G, B of primaries such that weighted sum of primaries is perceived the same as input spectrum
  - Get a metamer of input spectrum
- Grassmann's law: „color perception is linear“
  - Matching values for a sum of spectra with small spikes are the same as sum of matching values for the spikes
  - In the limit (spikes are infinitely narrow)

$$R = \int \bar{r}(\lambda)L(\lambda)d\lambda$$
$$G = \int \bar{g}(\lambda)L(\lambda)d\lambda$$
$$B = \int \bar{b}(\lambda)L(\lambda)d\lambda$$

  - Monochromatic matching curves $\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$

Summary

- Matching curves $\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$ define CIE RGB color space
  - Given spectrum, obtain CIE RGB values, or color "coordinates"
  - Every distinct color sensation can be represented using distinct CIE RGB tristimulus values
  - Many distinct spectra lead to same CIE RGB values (metamers)
- But
  - Not every color sensation can be produced with three primaries (negative values)
  - CIE RGB values directly relate to color perception only under standard viewing conditions used in matching experiment!

CIE color spaces

- CIE was not satisfied with range of RGB values for visible colors
  - Negative values
- Defined CIE XYZ color space via simple mathematical transformation
  http://en.wikipedia.org/wiki/CIE_1931_color_space#Definition_of_the_CIE_XYZ_color_space
- Most common color space still today

CIE XYZ color space

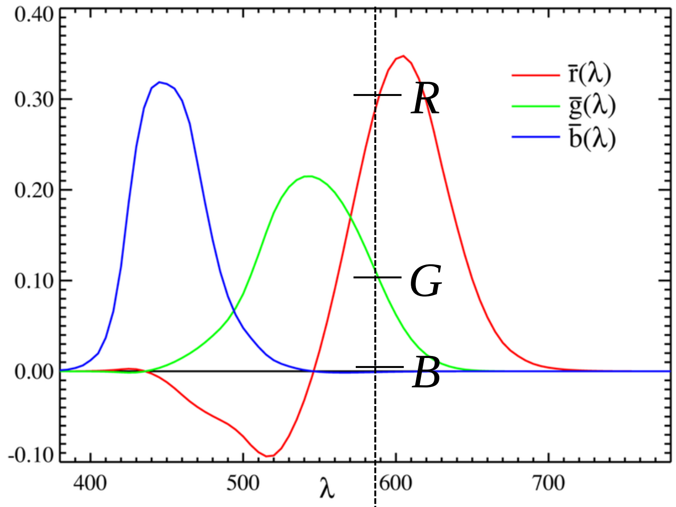
- Linear transformation of CIE RGB

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{b_{21}} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \frac{1}{0.17697} \begin{bmatrix} 0.49 & 0.31 & 0.20 \\ 0.17697 & 0.81240 & 0.01063 \\ 0.00 & 0.01 & 0.99 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

CIE XYZ <-- CIE RGB

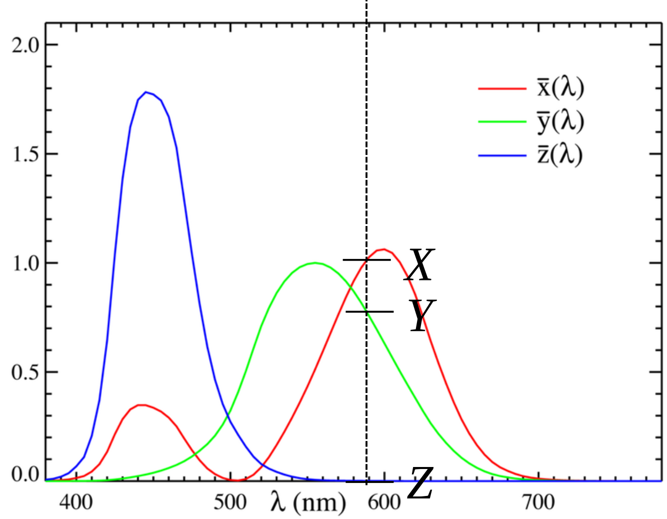
- Determined coefficients such that
  - Y corresponds to an experimentally determined brightness
  - No negative values in matching curves
  - White is XYZ=(1/3,1/3,1/3)

CIE XYZ color space



CIE RGB matching curves

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix}$$



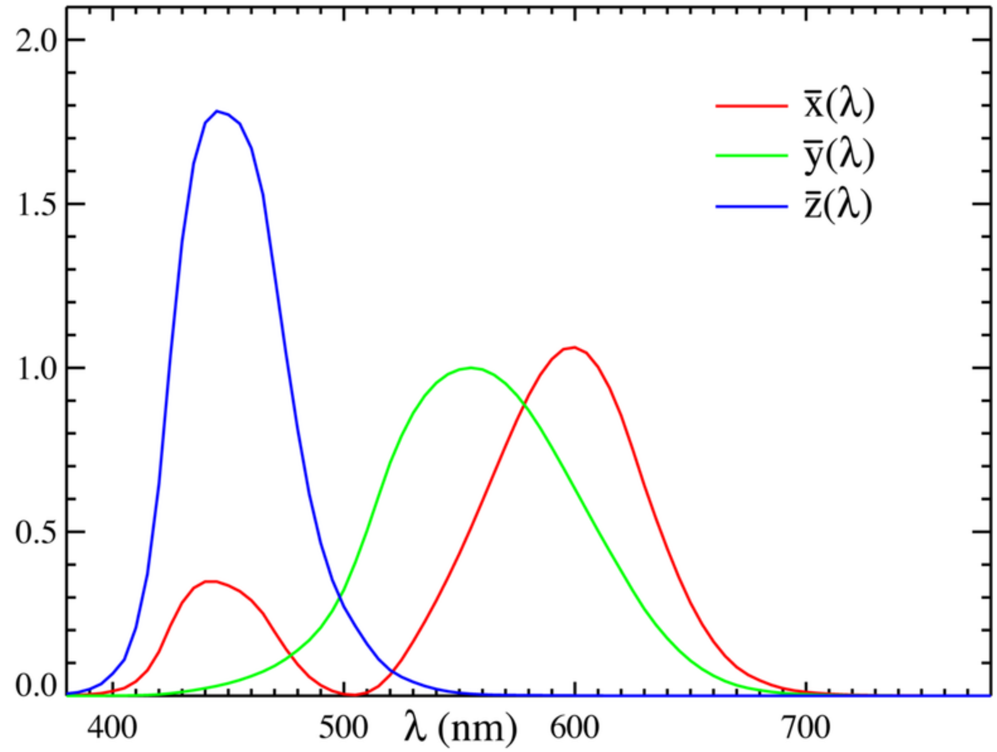
CIE XYZ matching curves

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{0.17697} \begin{bmatrix} 0.49 & 0.31 & 0.20 \\ 0.17697 & 0.81240 & 0.01063 \\ 0.00 & 0.01 & 0.99 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

CIE XYZ color space

Matching curves

- No corresponding physical primaries



Tristimulus values

- Always positive!

$$X = \int \bar{x}(\lambda) L(\lambda) d\lambda$$

$$Y = \int \bar{y}(\lambda) L(\lambda) d\lambda$$

$$Z = \int \bar{z}(\lambda) L(\lambda) d\lambda$$

Summary

- CIE XYZ obtained using linear transformation of CIE RGB
- Most commonly used quantification of colors used today
  http://en.wikipedia.org/wiki/CIE_1931_color_space#Definition_of_the_CIE_XYZ_color_space
- Most other standard color spaces defined by a one-to-one mapping from CIE XYZ
  - Not necessarily linear mapping, as between CIE XYZ and CIE RGB

Visualizing CIE XYZ

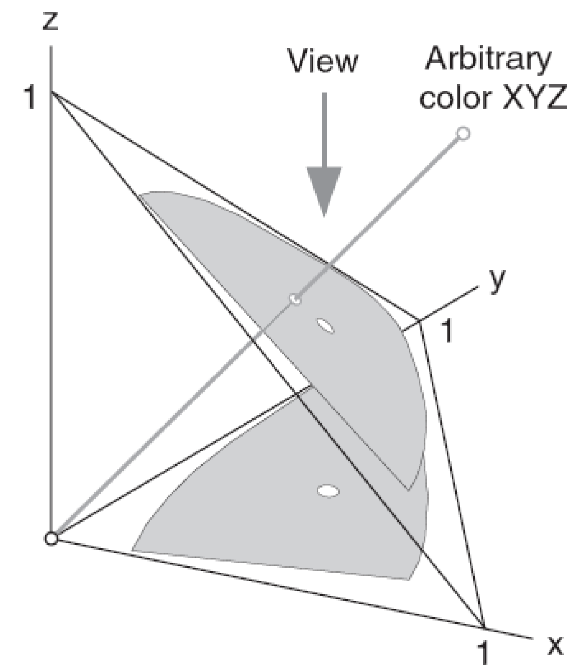
- Interpret XYZ as 3D coordinates
- Plot corresponding color at each point
- Many XYZ values do not correspond to visible colors

Chromaticity diagram

- Project to XYZ coordinates to 2D for more convenient visualization

$$x = \frac{X}{X+Y+Z} \quad y = \frac{Y}{X+Y+Z} \quad z = \frac{Z}{X+Y+Z}$$

- Drop z-coordinate



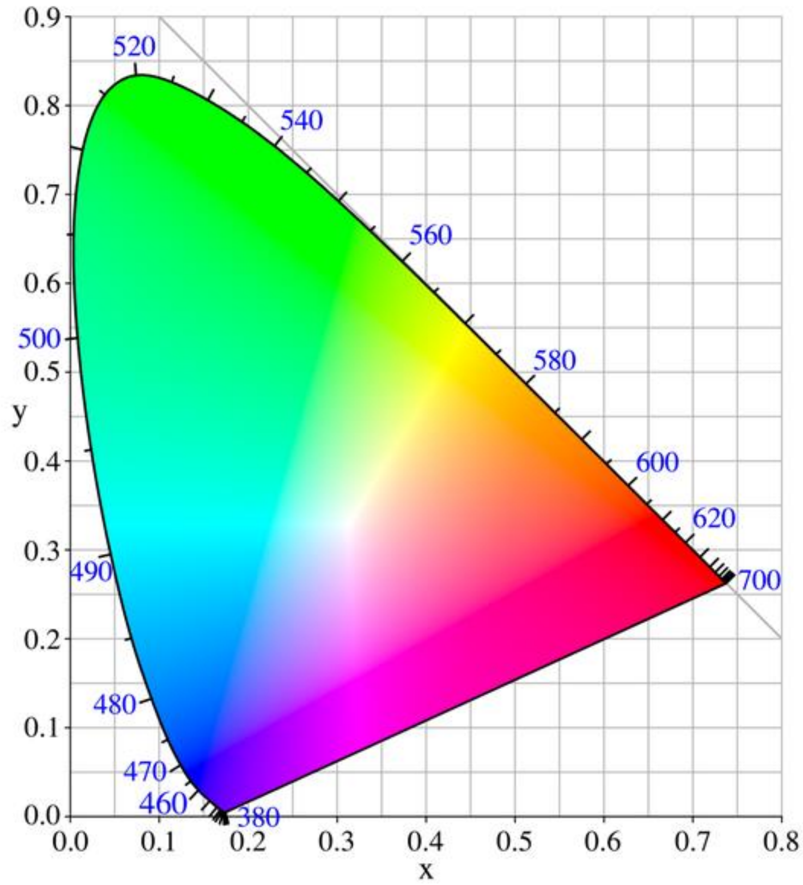
Chromaticity diagram

- Factor out luminance (perceived brightness) and chromaticity (hue)
  - x,y represent chromaticity of a color
  - Y is luminance
- CIE xyY color space
- Reconstruct XYZ values from xyY

$$X = \frac{Y}{y}x \qquad Z = \frac{Y}{y}(1 - x - y)$$

Chromaticity diagram

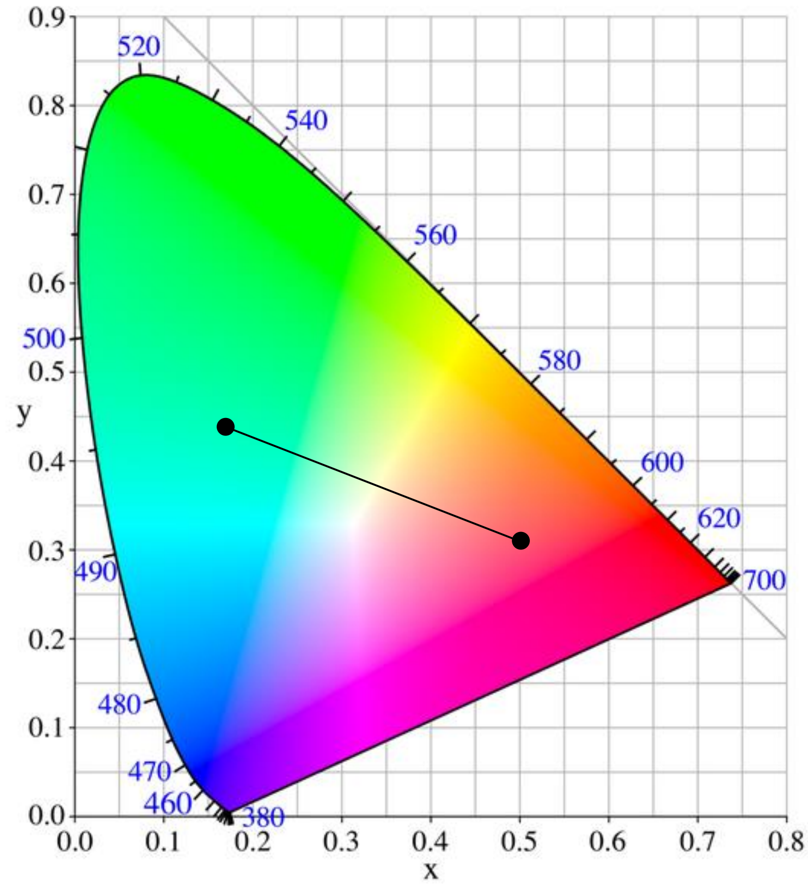
- Visualizes x,y plane (chromaticities)
- Pure spectral colors on boundary



Colors shown do not correspond to colors represented by (x,y) coordinates!

Chromaticity diagram

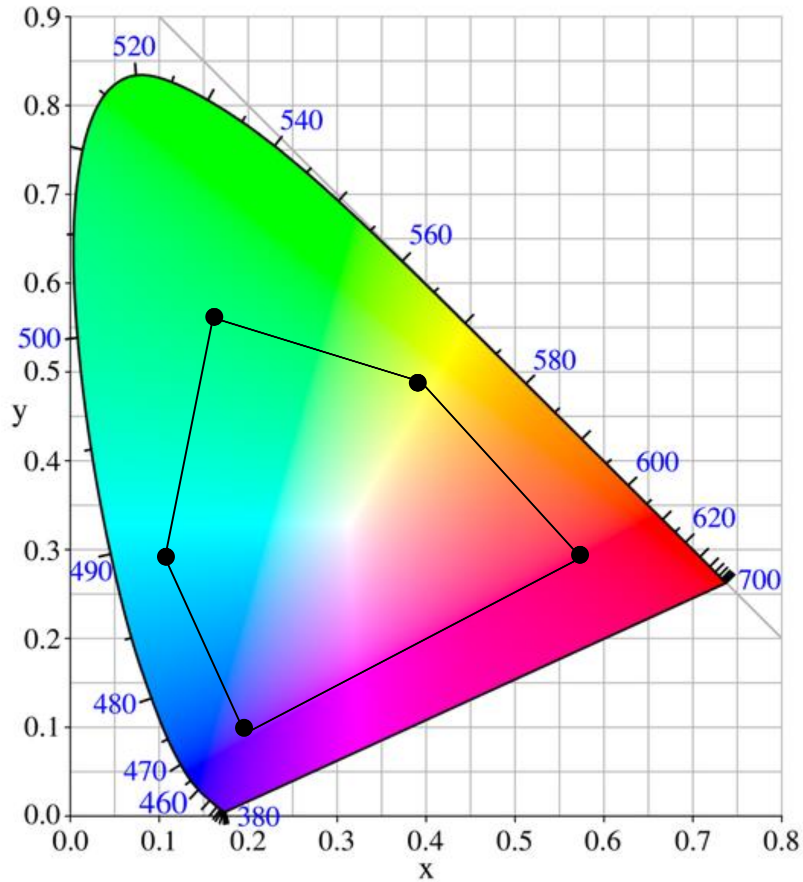
- Weighted sum of any two colors lies on line connecting colors



Colors shown do not correspond to colors represented by (x,y) coordinates!

Chromaticity diagram

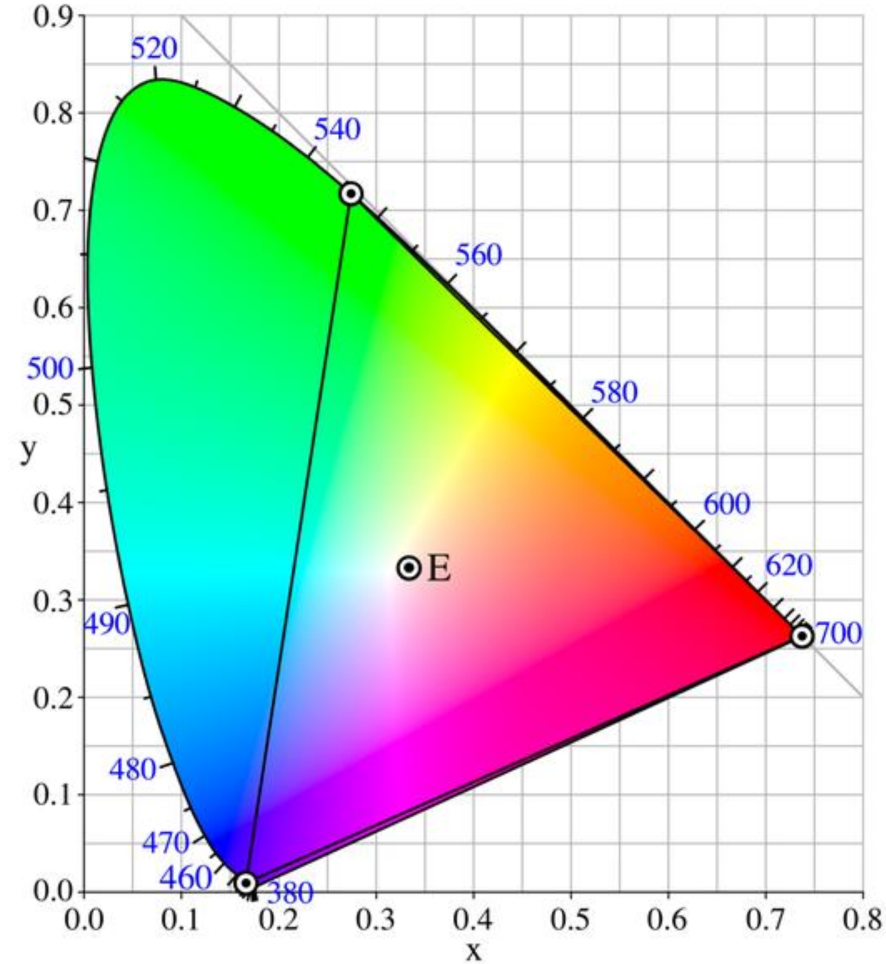
- Weighted sum of any number of colors lies in convex hull of colors (gamut)



Colors shown do not correspond to colors represented by (x,y) coordinates!

Gamut

- Any device based on three primaries can only produce colors within the triangle spanned by the primaries
- Does not cover all visible colors
- Points outside gamut correspond to negative weights of primaries



Gamut of CIE RGB primaries

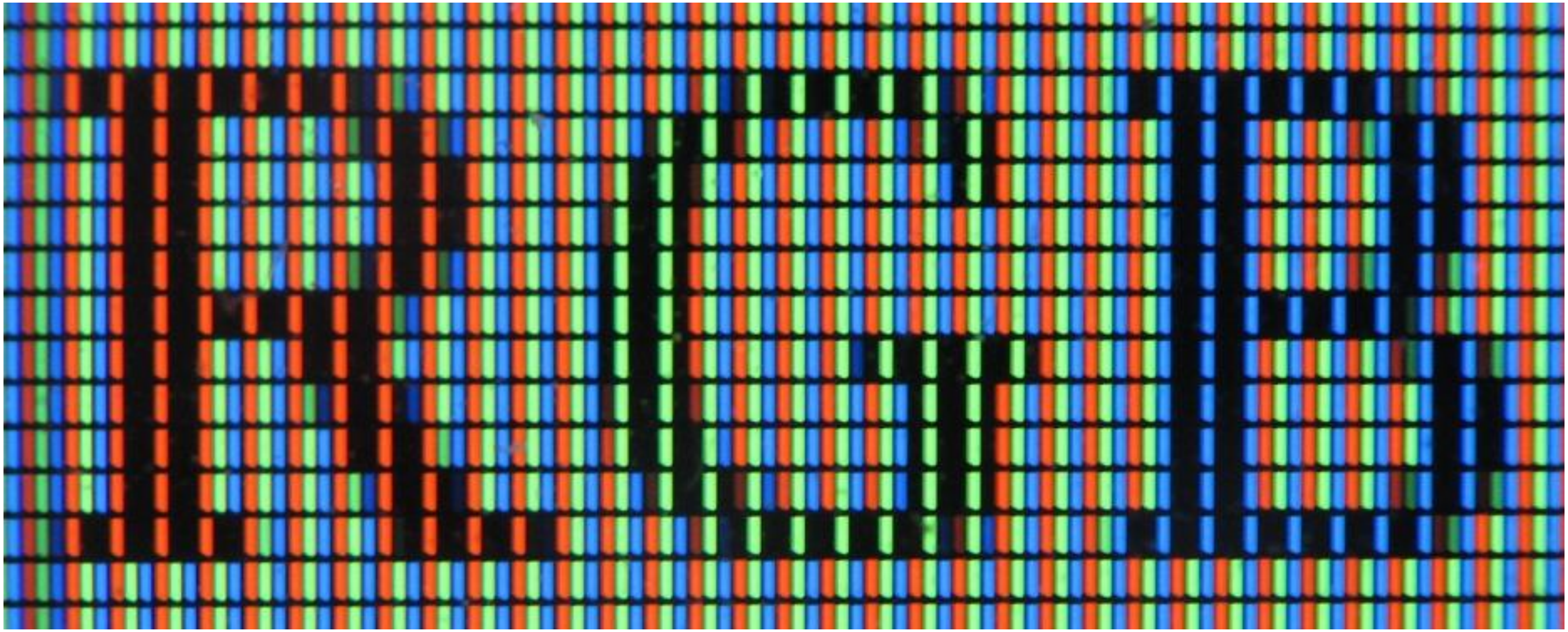
Today

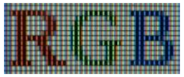
Color

- Physics background
- Color perception
- Color spaces
- Color reproduction on monitors
- Perceptually uniform color spaces

RGB monitors

- Each pixel consists of R,G,B subpixels





http://en.wikipedia.org/wiki/Liquid_crystal_display

RGB monitors

- Given rgb values, what color will your monitor produce?
  - I.e., what are the CIE XYZ or CIE RGB coordinates of the displayed color?
  - How are OpenGL RGB values related to CIE XYZ, CIE RGB?
- Usually you don’t know
  - OpenGL RGB ≠ CIE XYZ, CIE RGB

"Ideal" RGB monitor

- Assumptions
  - We know XYZ values for RGB primaries

$$(X_r, Y_r, Z_r)(X_g, Y_g, Z_g)\ (X_b, Y_b, Z_b)$$

  - Monitor is linear (emitted intensity proportional to input signal)
- Not true in practice!
- rgb input to monitor produces output

$$\begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix} = r \begin{bmatrix} X_r \\ Y_r \\ Z_r \end{bmatrix} + g \begin{bmatrix} X_g \\ Y_g \\ Z_g \end{bmatrix} + b \begin{bmatrix} X_b \\ Y_b \\ Z_b \end{bmatrix}$$

- Get a transformation matrix: monitor primaries define their own color space!

$$\begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix} = \begin{bmatrix} X_r & X_g & X_b \\ Y_r & Y_g & Y_b \\ Z_r & Z_g & Z_b \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

"Ideal" RGB monitors

- Given desired XYZ values, find rgb values by inverting transformation matrix

$$\begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix} \begin{bmatrix} X_r & X_g & X_b \\ Y_r & Y_g & Y_b \\ Z_r & Z_g & Z_b \end{bmatrix}^{-1} = \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

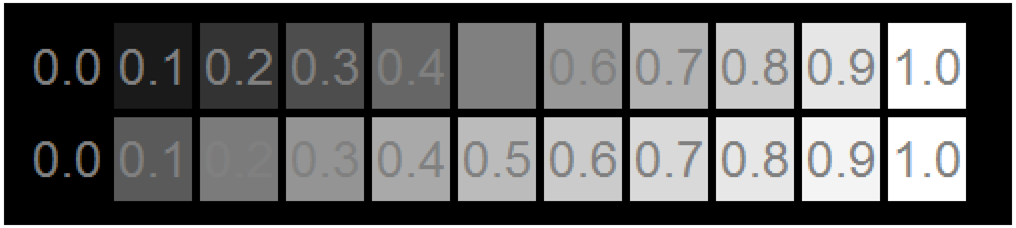
- Similar to change of coordinate systems for 3D points

In practice

- XYZ values for monitor primaries are usually not directly specified
  - Monitor brightness is adjustable
- “White” depends on illumination due to environment
- Perception of contrast is not linear
  - Perception more sensitive to absolute intensity differences in dark regions compared to bright regions

Linear perception

Linear intensity

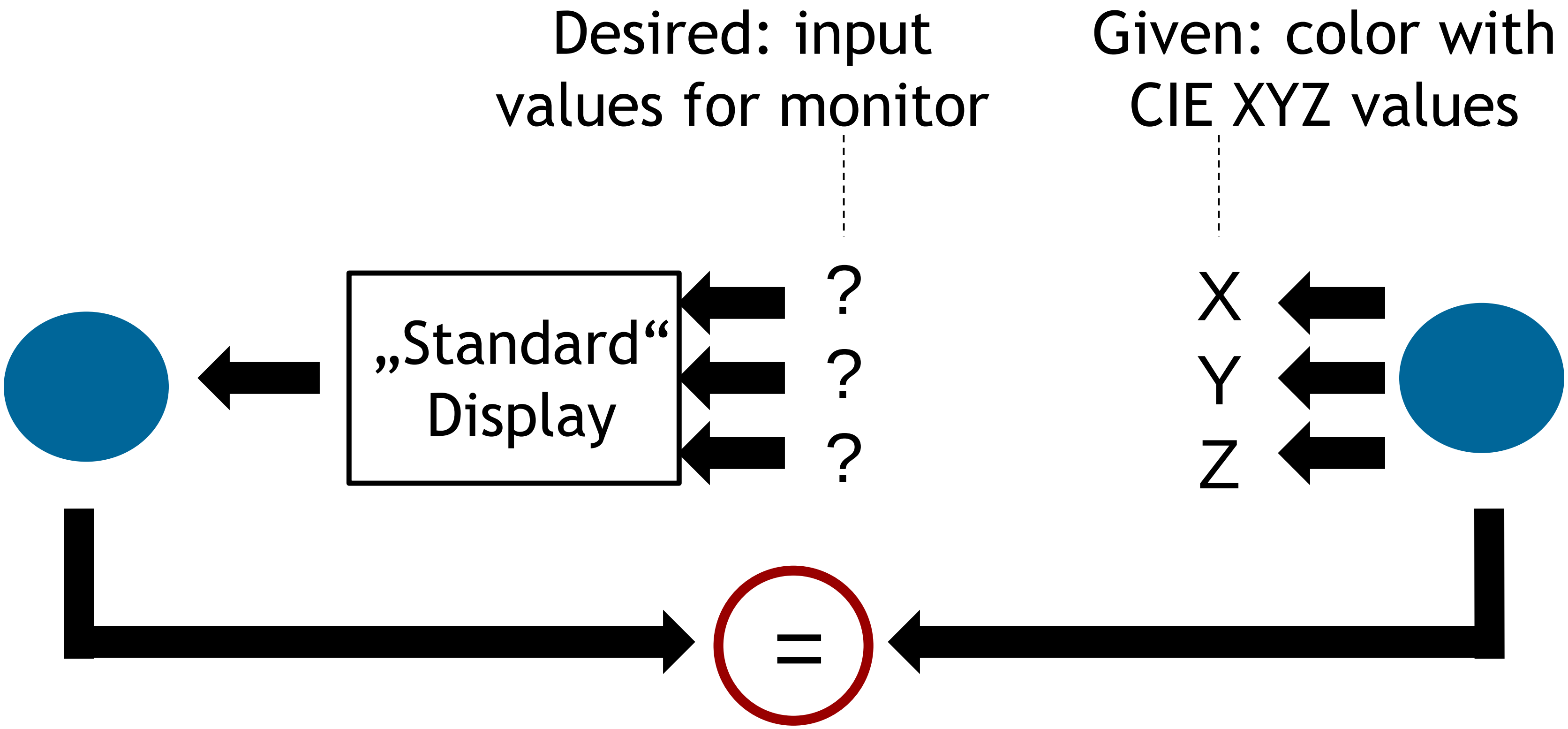


Gamma correction

- Non-linear transformation to take into account contrast sensitivity of human visual system (HVS)
  http://en.wikipedia.org/wiki/Gamma_correction

sRGB color space

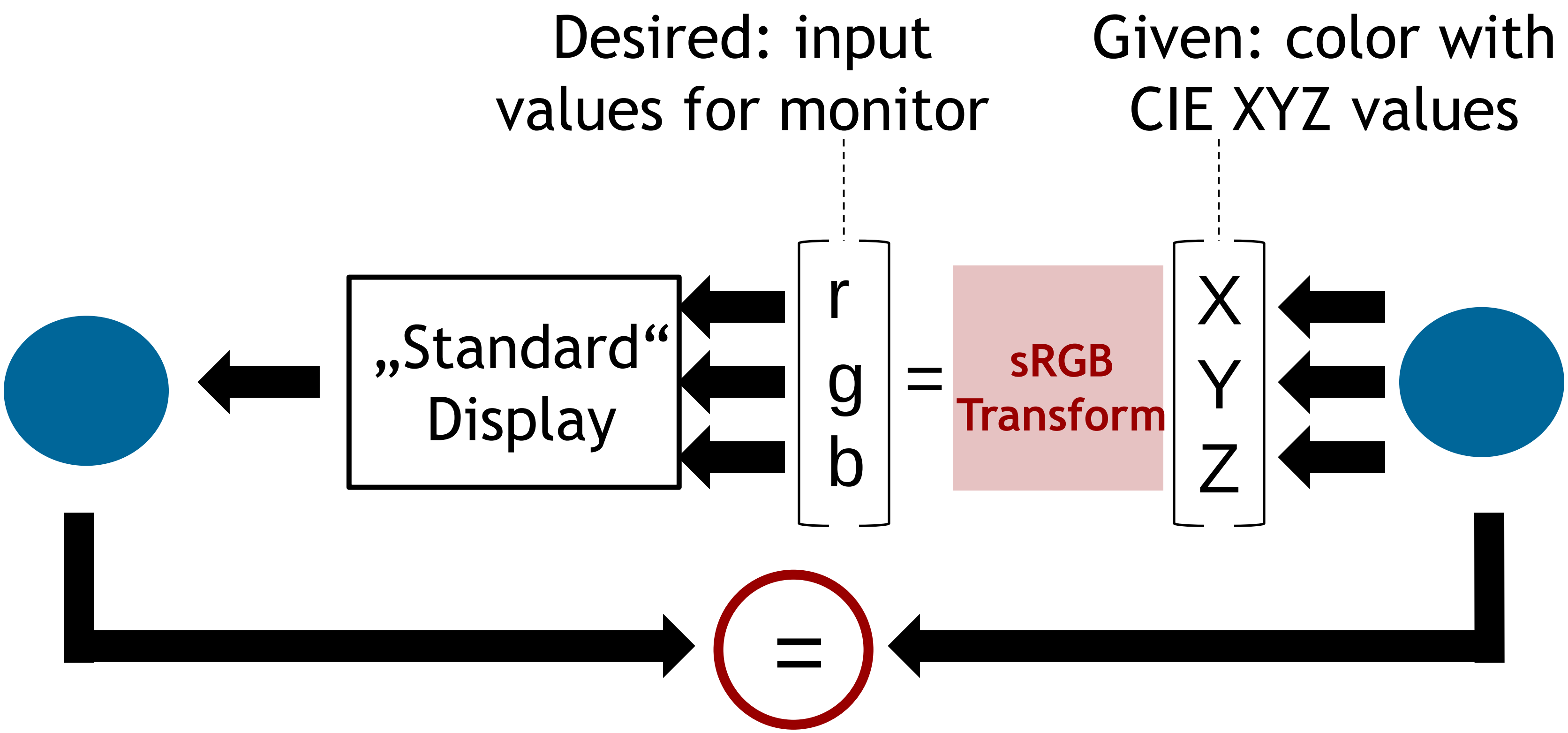
- Standard color space designed to describe "standard displays"
  http://en.wikipedia.org/wiki/SRGB



sRGB color space

- Standard color space designed to describe "standard displays"

http://en.wikipedia.org/wiki/SRGB



sRGB color space

http://en.wikipedia.org/wiki/SRGB_color_space

- CIE XYZ to sRGB transformation

1. Linear mapping

$$\begin{bmatrix} R_{\mathrm{linear}} \\ G_{\mathrm{linear}} \\ B_{\mathrm{linear}} \end{bmatrix} = \begin{bmatrix} 3.2410 & -1.5374 & -0.4986 \\ -0.9692 & 1.8760 & 0.0416 \\ 0.0556 & -0.2040 & 1.0570 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

2. Gamma correction

$$C_{\mathrm{srgb}} = \begin{cases} 12.92 C_{\mathrm{linear}}, & C_{\mathrm{linear}} \le 0.0031308 \\ (1+a) C_{\mathrm{linear}}^{1/2.4} - a, & C_{\mathrm{linear}} > 0.0031308 \end{cases}$$

$$C_{linear} = \{R_{linear}, G_{linear}, B_{linear}\}$$

sRGB color space

- Defined by a transformation of XYZ values
- „Given XYZ values, sRGB transformation maps XYZ to RGB input values for a standard monitor, such that displayed color matches original XYZ values“
- Based on assumptions about „standard monitor“
- Includes a non-linear gamma transformation to compensate for non-linear contrast sensitivity of HVS

Color calibration

- Color reproduction on consumer monitors often less than perfect
  - Same RGB values on one monitor look different than on another
- Need color calibration
  - Derive device specific transformation of color space
  - Based on measurements of input-output relation of specific device
  - Standard for digital publishing, printing, photography
  - Consumers do not seem to care
- Color management usually done by operating system
  - E.g., Windows Color System (Vista)
  - Needs appropriate “color profiles” (calibration data)

Display calibration

Today

Color

- Physics background
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Perceptually uniform color spaces

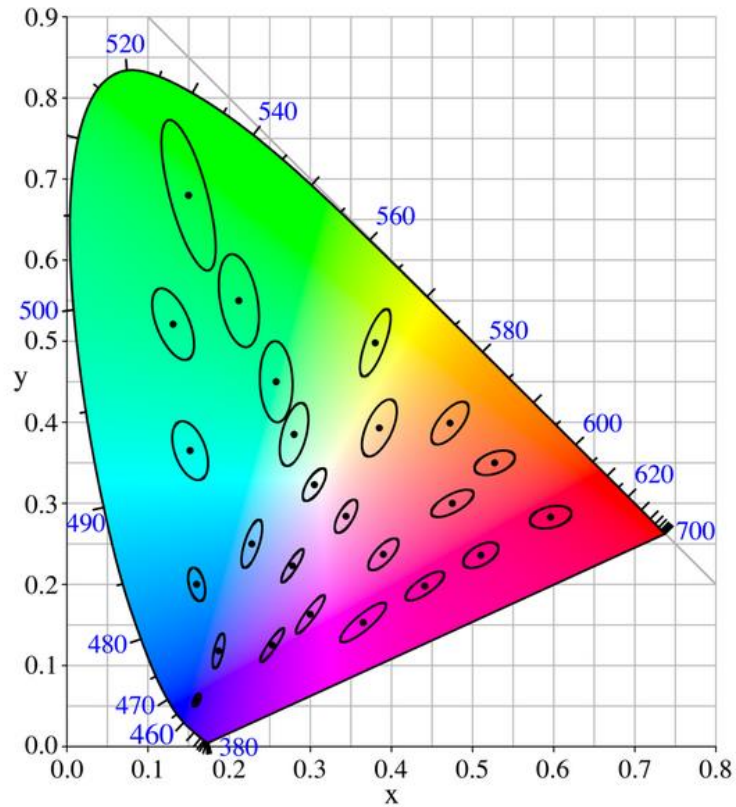
Definition

Euclidean distance between color coordinates corresponds to perceived difference

- CIE RGB, XYZ are not perceptually uniform
  - Euclidean distance between RGB, XYZ coordinates does not correspond to perceived difference

MacAdam ellipses

- Experiment (1942) to identify regions in CIE xy color space that are perceived as the same color
- Found elliptical areas, MacAdam ellipses
  - Ellipse boundaries indicate just noticeable difference (jnd)
    http://en.wikipedia.org/wiki/Just-noticeable_difference
- In perceptually uniform color space, each point on jnd ellipse should be at same distance to center
  - Use distortion of plane such that ellipses become circles

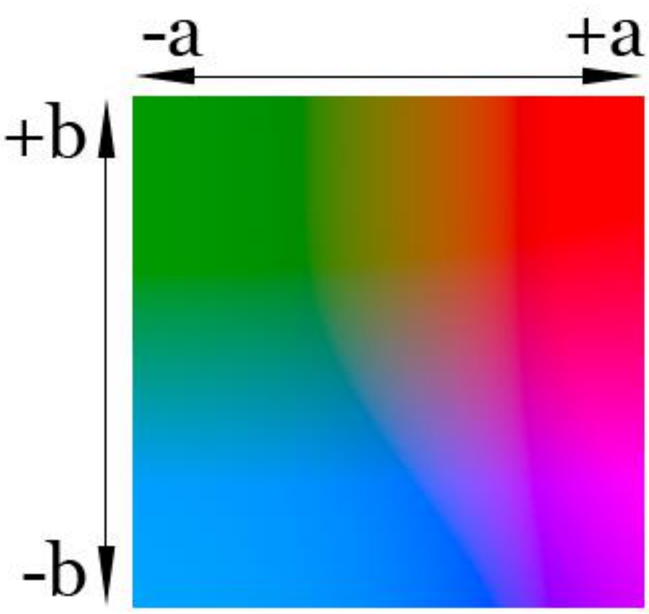


MacAdam ellipses

CIE L*,a*,b* (CIELAB)

- Most common perceptually uniform color space
  - L* encodes lightness
  - a* encodes position between magenta and green
  - b* encodes position between yellow and blue
- Conversion between CIE XYZ and CIELAB is non-linear

http://en.wikipedia.org/wiki/Lab_color_space



CIELAB gamut
Displayed colors do not match CIELAB colors

Next time

- Shading