1. Just to work with it, given points $P_0 = (90, 120)$ and $P_1 = (150, 180)$, give the equations for a parametric line through both points (starts at $P_0$, ends at $P_1$). Also give the midpoint of the segment.

$$x(t) = 90 + 60t$$

$$y(t) = 120 + 60t$$

Midpoint:

$$x(0.5) = 90 + 60 \times 0.5 = 120$$

$$y(0.5) = 120 + 60 \times 0.5 = 150$$

2. Given the following two parametric curves, validate that they satisfy the associated implicit equations by substitution and reduction to identity. Assume for both the range of $t$ is $(-\infty, \infty)$.

**Parabola solution:** substitute parametric equations into the implicit equation and derive identity

$$x^2 - 4ay = 0$$

$$\Rightarrow (2at)^2 - 4aat^2 = 0$$

$$\Rightarrow 4a^2t^2 - 4a^2t^2 = 0$$

$$\Rightarrow 0 = 0$$

Another solution to is solve $x(t) = 2at$ for $t$, giving $t = \frac{x}{2a}$, and then substitute for $t$ in $y(t) = at^2$ and show you get the implicit equation.

**Hyperbola solution.**

$$\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$$

$$\Rightarrow \left(\frac{a\sec t}{a}\right)^2 - \left(\frac{b\tan t}{b}\right)^2 = 1$$

$$\Rightarrow \left(\sec t \right)^2 - \left(\tan t \right)^2 = 1$$

$$\Rightarrow \sec^2 t - \tan^2 t = 1$$

$$\Rightarrow \sec^2 t = 1 + \tan^2 t \cdot \therefore$$

3. For the parabola in question (2), would any monotonically increasing function of $t$ ($t^2, \sqrt{t}$, etc) be valid in the parametric equations? *Yes – it changes the rate that $t$ sweeps out the curve, but not the shape. But, it might change the portions of the curve that are drawn – if $t$ in the range $[-1,1]$, then $t^2$ is in the range $[0,1]$. E.g., would the parametric equations below also be valid? *Yes – the parametric equations based on the square root also satisfy the the implicit equation. However, the range drawn may be only part of the full curve.

$$x^2 - 4ay = 0$$

$$\Rightarrow (2a\sqrt{t})^2 - 4a(a\sqrt{t}^2) = 0$$

$$\Rightarrow 4a^2t - 4a^2t = 0$$

$$\Rightarrow 0 = 0$$
What would happen if you used non-increasing functions of t, like cos or sin?

If you substitute a non-increasing function then the values in the parametric equation will cycle, or repeat, and possibly not sweep out the entire curve because the range is limited. For example, cos will cycle between -1 and 1, so a curve that requires t outside that range might partially draw the curve.

4. Sketch the 2D implicit curve for the function \( f : \mathbb{R}^2 \rightarrow \mathbb{R} \) where: \( f(x, y) = |x| + |y| - 1 \)

Label the 2D coordinates as appropriate.

Does this function satisfy the inside/outside convention for implicit curves? Why? (The inside/outside convention is that the implicit function is negative inside the curve, positive outside, and zero right on the curve.)

Yes. Points inside have negative values.
- \( f(0,0) = -1, f(0,2,0.2) = -0.6 \)
- Points outside have positive values
- \( f(2,1) = 2, f(2,0) = 2 \)

5. The Yin Yang symbol can be drawn as a set of parametric curves. In particular, the outlines of the nested “tadpole” regions can be drawn by drawing in sequence a set of curves, and then obviously the two smaller circles are parametric circles. Assuming you’re drawing the symbol centered at zero with a radius of 1, and are using the standard parametric circle equations \( x(t) = x_{center} + radius \times \cos(t) \), \( y(t) = y_{center} + radius \times \sin(t) \)

The symbol can be decomposed into four subshapes: the white tadpole, the black tadpole, the black circle and the white circle. Clockwise is cw, counterclockwise is ccw.

Small black circle: full circle with t in 0 to 2\( \pi \) and center of (0,0.5) and radius of (about) 0.2
Small white circle: full circle with t in 0 to 2\( \pi \) and center of (0,-0.5) and radius of (about) 0.2
White tadpole: Starting at top point (0,1) draw in sequence
- Circle arc of center (0,0), radius 1, and t from \( \pi/2 \) to 3\( \pi/2 \) (cw)
- Circle arc of center (0,-0.5), radius 0.5, and t from 3\( \pi/2 \) to \( \pi/2 \) (ccw, negative increment)
- Circle arc of center (0,0.5), radius 0.5, and t from \( 3\pi/2 \) to 5\( \pi/2 \) (cw, positive increment)
Black tadpole: Starting at bottom point (0,-1) draw in sequence
- Circle arc of center (0,0), radius 1, and t from 3\( \pi/2 \) to 5\( \pi/2 \) (cw)
- Circle arc of center (0,0.5), radius 0.5, and t from 5\( \pi/2 \) to 3\( \pi/2 \) (cw, negative increment)
- Circle arc of center (0,0.5), radius 0.5, and t from \( \pi/2 \) to 3\( \pi/2 \) (cw, positive increment)

Full credit was given for correctly giving each arc. Once you have the arcs there are two ways to combine for the symbol. One is to draw each arc in sequence, so for example you draw the white tadpole as a sequence of three subarcs. A second way is to draw shapes on top of each other, starting with a white full circle, then a black half circle, then two smaller full circles of black and white, and finally the two smallest circles of black and white.