■ EXAMPLE 4.5.8 Characterize a patch.

Find **a**, **b**, and C that create a square patch of length 4 on a side centered at t
origin and parallel to the x, z-plane.

SOLUTION:

The corners of the patch are at $(2, 0, 2)$, $(2, 0, -2)$, $(-2, 0, 2)$, and $(-2, 0, -2)$. Choose a corner, say $(2, 0, -2)$, for C. Then **a** and **b** each have length 4 and are parallel to either t x- or the z-axis. Choose $\mathbf{a} = (-4, 0, 0)$ and $\mathbf{b} = (0, 0, 4)$.

PRACTICE EXERCISE

4.5.9 Find a patch.

Find point C and some vectors **a** and **b** that create a patch having the four corne $(-4, 2, 1)$, $(1, 7, 4)$, $(-2, -2, 2)$, and $(3, 3, 5)$.

4.6 FINDING THE INTERSECTION OF TWO LINE SEGMENTS

If the facts don't fit the theory, change the facts.

Albert Einstei
(1879–1955)

We often need to compute where two line segments in 2D space intersect. It appear in many other tasks, such as determining whether or not a polygon is simple. Its solution will illustrate the power of parametric forms and dot products. We will be restricting ourselves here to finding the intersection of two lines when they ar represented parametrically.

The Problem: Given two line segments, determine whether they intersect, and i they do, find their point of intersection.

Suppose one segment has endpoints A and B and the other segment has endpoints C and D. As shown in Figure 4.32, the two segments can be situated in man different ways. They can miss each other (a and b), overlap in one point (c and d), or even overlap over some region (e). They may or may not be parallel. We need an organized approach that handles all of these possibilities.

Every line segment has a **parent line**, the infinite line of which it is part. Unless two parent lines are parallel, they will intersect at some point in 2D. We first locate this point

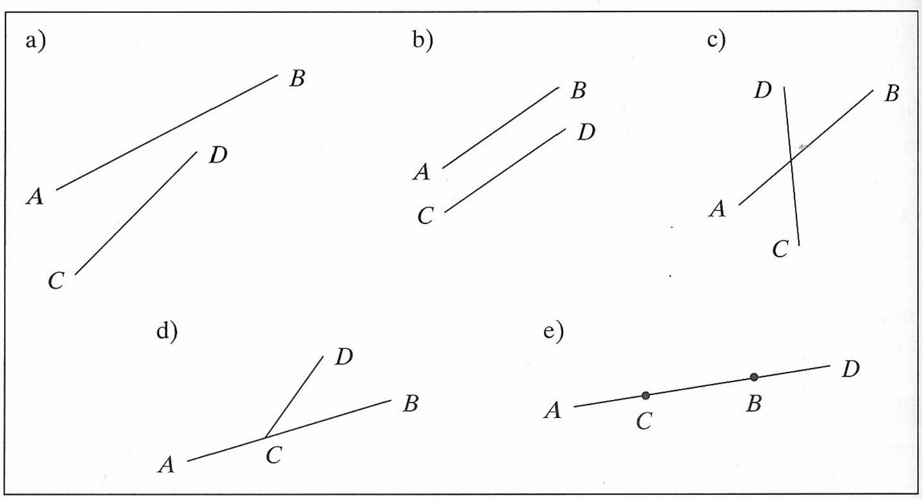

FIGURE 4.32 Many cases for two line segments.

We set up parametric representations for each of the line segments in question. Call AB the segment from A to B. Then

$$AB(t) = A + \mathbf{b}t \tag{4.45}$$

where for convenience we define $\mathbf{b} = B - A$. As t varies from 0 to 1, each point on the finite line segment is crossed exactly once.

Similarly we call the segment from C to D by the name CD, and give it parametric representation (using a *different* parameter, say, u):

$$CD(u) = C + \mathbf{d}u,$$

where $\mathbf{d} = D - C$. We use different parameters for the two lines, t for one and u for the other, in order to describe different points on the two lines independently. (If the same parameter were used, the points on the two lines would be locked together.)

For the parent lines to intersect there must be specific values of t and u for which the two equations above are equal:

$$A + \mathbf{b}t = C + \mathbf{d}u$$

Defining $\mathbf{c} = C - A$ for convenience, we can write this condition in terms of three known vectors and two unknown parameter values:

$$\mathbf{b}t = \mathbf{c} + \mathbf{d}u \tag{4.46}$$

This provides two linear equations in two unknowns, similar to Equation (4.22). They can be solved in the usual manner.

PRACTICE EXERCISES

4.6.1 The algorithm for determining the intersection

Write the routine `segIntersect()` that would be used in the context: `if(segIntersect(A, B, C, D, &InterPt)) <do something>`.

It takes four points representing the two segments, and returns 0 if the segments do not intersect, and 1 if they do. If they do intersect, the location of the intersection is placed in `interPt`. It returns −1 if the parent lines do not intersect.

4.6.2 Testing the simplicity of a polygon

Recall that a polygon P is simple if there are no edge intersections except at the endpoints of adjacent edges. Fashion a routine `int isSimple(Polygon P)` that takes a brute-force approach and tests whether any pair of edges of the list of vertices of the polygon intersect, returning 0 if so, and 1 if not so. (`Polygon` is some suitable class for describing a polygon.) This is a simple algorithm but not the most efficient one. See [Moret91] and [Preparata85] for more elaborate attacks that involve some sorting of edges in x and y.

4.6.3 Line segment intersections

For each of the following segment pairs, determine whether the segments intersect, and if so, where.

1. $A = (1, 4)$, $B = (7, 1/2)$, $C = (7/2, 5/2)$, $D = (7, 5)$.
2. $A = (1, 4)$, $B = (7, 1/2)$, $C = (5, 0)$, $D = (0, 7)$.
3. $A = (0, 7)$, $B = (7, 0)$, $C = (8, -1)$, $D = (10, -3)$. ■