

# Please Fill Out All of Your Courses Teaching Evals

December 31, 2024

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- 6) The biggest problem we have for all of the above is when not that many students fill them out. Hence
- 7) **Please Fill Out the Teaching Evals in All of your Courses**

# Topics Not Covered in Grad Ramsey 2022

**Exposition by William Gasarch**

December 31, 2024

# We Didn't Cover $X$ Because...

What topics in Ramsey theory didn't we cover?

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- ▶ Too hard for Bill.
- ▶ Some combination of the above.

# Could Have Covered: VDW

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# Canonical VDW

**Can VDW** For all  $k$  there exists  $W = W(k)$  such that for any  $\text{COL}: [W] \rightarrow [\omega]$  there exists  $a, d$  such that either

$a, a + d, \dots, a + (k - 1)d$  are all the same color

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**Research** Better bounds on Can VDW Numbers.

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I could have proven this in class and might next time I teach it.

**Research** The proof gives VDW-like bounds. Hard NT gives better bounds. Get better bounds in elementary way.

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Certainly could have done this and have in past semesters.

# Folkman's Thm

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- ▶ Some subset of the  $a_i$ 's sums to 0.
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Great thm, nice proof. Might cover it in the future.

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- ▶ Caution: Some of this may be known.

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- ▶ I've taught before and could teach again.

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This is Roth's proof done with the ideas showing and the computation rightly put into the background.
- ▶ **Research** Get better bounds: How big a subset of  $\{1, \dots, 1000\}$  before guaranteed a 3-AP? 4-AP? etc.

# A Stupid App of Schur's Thm to Number Theory

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**FLT** For all  $n \geq 3$  there does not exist  $x, y, z \in \mathbb{N}$  such that  
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Gasarch proved:

**Thm** (Schur's Thm + FLT(4)) implies there are an infinite number  
of primes. [https://www.cs.umd.edu/users/gasarch/  
COURSES/858/S20/notes/schurflt.pdf](https://www.cs.umd.edu/users/gasarch/COURSES/858/S20/notes/schurflt.pdf)

# Rado's Theorem over the Reals

## Vote

For all  $\text{COL}: \mathbb{R} \rightarrow \mathbb{N}$  there exists  $w, x, y, z$  all the same color:

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- ▶ TRUE
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Proven by Erdos. Write up by Fenner and Gasarch is here:

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### Sample Thm

$$R(C_k) = \begin{cases} 6 & \text{if } k = 3 \text{ or } k = 4 \\ 2k - 1 & \text{if } k \geq 5 \text{ and } k \equiv 1 \pmod{2} \\ \frac{3k}{2} - 1 & \text{if } k \geq 6 \text{ and } k \equiv 0 \pmod{2} \end{cases} \quad (1)$$

## Other Ramsey Numbers

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### Sample Thm

$$R(C_k) = \begin{cases} 6 & \text{if } k = 3 \text{ or } k = 4 \\ 2k - 1 & \text{if } k \geq 5 \text{ and } k \equiv 1 \pmod{2} \\ \frac{3k}{2} - 1 & \text{if } k \geq 6 \text{ and } k \equiv 0 \pmod{2} \end{cases} \quad (1)$$

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- ▶ For every result of this type see <https://www.combinatorics.org/files/Surveys/ds1/ds1v15-2017.pdf>

# Research Projects

- ▶ Actually FIND the colorings.
- ▶ Simplify or unify the proofs
- ▶ **Ramsey Games** Example: Parameter  $k, n$ . Players RED and BLUE alternate coloring the edges of  $K_n$ . RED goes first. The first player to get a  $C_k$  in their color wins.
  1. For which  $n$  does RED have a winning strategy?
  2. Design an ML to play this well (my REU project)
  3. EVERY thm in Ramsey Thm (and the VDW part) can be made into a game and lead to research projects.



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**Research** Use their technique on other Ramsey problems.

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- ▶ Do we really need more Can Ramsey in the course?

# Large Can Ramsey

The following is well known; however, I may be the first person to write down the proof.

<http://www.cs.umd.edu/~gasarch/COURSES/858/S20/notes/canlarge.pdf>

**Thm** For all  $k$  there exists  $n = n(k)$  such that for all  
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**Research** Get the bound in terms of  $LR_3$  or lower.



## $a$ -ary Can Ramsey

**Thm** For all  $a, k \in \mathbb{N}$  there exist  $C = C(a, k)$  such that for all  $\text{COL}: \left[ \binom{[C]}{a} \right] \rightarrow [\omega]$  there exists a set  $H$ ,  $|H| = k$  and  $1 \leq i_1 < \dots < i_L \leq a$  such that for all  $p_1 < \dots < p_a \in H$  and  $q_1 < \dots < q_a \in H$

$\text{COL}(p_1, \dots, p_a) = \text{COL}(q_1, \dots, q_a)$  iff  $(p_{i_1}, \dots, p_{i_L}) = (q_{i_1}, \dots, q_{i_L})$

- ▶ Similar to the proof on graphs, but messier.
- ▶ *On canonical Ramsey numbers for coloring three-element sets* by Lefmann and Rodl behind paywalls, lost to humanity.
- ▶ Optimal results due to Shelah:  
<https://arxiv.org/abs/math/9509229> A hard read.

**Research** Give easier proofs of bounds.

# Could have Covered: Euclidean Ramsey Theory

**Exposition by William Gasarch**

December 31, 2024

# Euclidean Ramsey Theory

**Sample Thm** Let  $T$  be a triangle with a 30, 90, or 150 degree angle. For every 2-coloring of  $\mathbb{R}^2$  there exists three points that form triangle  $T$  (note- actually form  $T$ , not just similar to  $T$ ) that are monochromatic.

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- ▶ For more:  
<https://www.csun.edu/~ctoth/Handbook/chap11.pdf>

# Results Bill Likes But Would be Hard to Teach:VDW

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- ▶ Alice is Poly time and she has  $x$ ,  $|x| = n$ .
- ▶ Bob is all powerful and he has nothing.
- ▶ They cooperate to determine if  $x \in L$ . Alice could just send Bob  $x$ . That takes  $n$  bits.

## App of 3-Free Sets to Complexity Theory Cont

Let  $L$  be the set of all 3-colorable graphs (or any NPC graph problem). Note size is  $O(n^2)$ . Is there a protocol for Alice and Bob in  $O(n^{2-\epsilon})$  bits for some  $\epsilon > 0$ ?

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- ▶ **Research** Come up with an elementary proof.

# Results Bill Likes But Would be Hard to Teach: Ramsey

**Exposition by William Gasarch**

December 31, 2024

# Results from Logic

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**Thm** For every computable COL:  $\binom{\mathbb{N}}{a} \rightarrow [c]$  there is a  $\Pi_a$ -homogenous set. There is a computable coloring such that no homog set is  $\Sigma_a$ .

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**Thm** Fix  $k$ . For large  $n$ , for all 2-colorings of  $K_n$  there exists  $\frac{n^2}{4^{k^2(1+o(1))}}$  mono  $K_k$ 's.

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- ▶ **Research** Look at col  $G$  to get mono  $H$  for other  $G$  and  $H$ .

# Results Bill Likes But Would be Hard to Teach: Complexity

**Exposition by William Gasarch**

December 31, 2024

# Complexity: $\Pi_2^P$ Completeness of Arrow

**Def**  $G \rightarrow (H_1, H_2)$  means that for every 2-coloring of the edges of  $G$  there is either a **RED**  $H_1$  or a **BLUE**  $H_2$ .

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Marcus Schaefer proved the following.

**Thm**  $\{(G, H_1, H_2) : G \rightarrow (H_1, H_2) \text{ is } \Pi_2^P\text{-complete.}$

See <http://www.cs.umd.edu/~gasarch/COURSES/858/S20/notes/npramsey.pdf>



# Complexity: NP-Completeness of Grid Extension

*Grid Color Extension (GCE)* is the set of tuples  $(n, m, c, \chi)$  such that the following hold:

- ▶  $n, m, c \in \mathbb{N}$ .  $\chi$  is a partial  $c$ -coloring of  $[n] \times [m]$  that is rectangle-free.
- ▶  $\chi$  can be extended to a rectangle-free coloring of  $[n] \times [m]$ .

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**Thm** (Apon, Gasarch, Lawler) *GCE* is NP-complete

<https://arxiv.org/pdf/1205.3813.pdf>

# Complexity: NP-Completeness of Grid Extension

*Grid Color Extension (GCE)* is the set of tuples  $(n, m, c, \chi)$  such that the following hold:

- ▶  $n, m, c \in \mathbb{N}$ .  $\chi$  is a partial  $c$ -coloring of  $[n] \times [m]$  that is rectangle-free.
- ▶  $\chi$  can be extended to a rectangle-free coloring of  $[n] \times [m]$ .

**Thm** (Apon, Gasarch, Lawler) *GCE* is NP-complete

<https://arxiv.org/pdf/1205.3813.pdf>

Jacob Proofread This!

# Complexity: Long Proofs Required

**Def** Resolution proofs are a proof system to show that a Boolean Formula is NOT satisfiable. It is of interest to find a class of non-satisfiable formulas  $\phi_n$  that require (say)  $(1.5)^n$  long Res Proofs.

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Lauria, Pudlak, Rodl, Thapen proved:

**Thm** For appropriate  $c$ , any resolution proof for  $\phi_{n,c}$  requires length  $n^{\Omega(\log n)}$ .

<https://arxiv.org/pdf/1303.3166.pdf>

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**Research** What we **really** want is evidence that computing  $R(k)$  is hard. These results do not really do that. Maybe you can!

**Research** Look at the above results for particular cases and see if easier.

# Results Bill Does Not Care About But Should:VDW

**Exposition by William Gasarch**

December 31, 2024

# Rado's Thm for Matrices

**Rado's Thm** Let  $a_1, \dots, a_k \in \mathbb{Z}$ . TFAE

- ▶ Some subset of the  $a_i$ 's sums to 0.
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For a statement of the thm see the Wikipedia entry.

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This is someone else's slides on it. So I REALLY could have covered it!

https:

[//www.ti.inf.ethz.ch/ew/courses/extremal04/razen.pdf](https://www.ti.inf.ethz.ch/ew/courses/extremal04/razen.pdf)

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[https://www.ams.org/journals/tran/1971-159-00/  
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# Results Bill Does Not Care About But Should: Ramsey

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- ▶ Ramsey Cardinals on Next Slide.



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**Def**  $\kappa$  is **inaccessible** if  $\alpha < \kappa \implies 2^\alpha < \kappa$ .

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**Vote:** YES, NO, or OTHER.

**Thm** If  $\kappa$  is Ramsey then  $\kappa$  is inaccessible. (The converse is ind of ZFC but reasons to think its false.)

# Results Bill May One Day Learn But Still too Hard for the Students

**Exposition by William Gasarch**

December 31, 2024

# Ramsey's Thm with control of the differences

**Thm** For all  $c, k$  and for all order types  $\eta$  there exists  $N = N(c)$  such that for all COL:  $[N] \rightarrow [c]$  there exists a homog set  $a_1 < \dots < a_k$  such that

$$(a_2 - a_1, a_3 - a_2, \dots, a_n - a_{n-1})$$

are all distinct and are in order type  $\eta$ .

- ▶ First proven by Noga Alon and Jan Pach using VDW, so bounds on  $N(c)$  are large. Later Noga Alon, Alan Stacey, and Saharon Shelah got an iterated exp bound. None of this is written down anywhere.
- ▶ In 1995 Saharon Shelah got double exp bounds <https://arxiv.org/pdf/math/9502234.pdf>
- ▶ Shelah's paper is hard. I'm looking for easier proof of weaker results.



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**Research** Easier Proof.

**Caveat** There is a proof of Sz thm for Hales-Jewitt which is said to be elementary.

<https://arxiv.org/abs/0910.3926>

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**Research** Look for the AP's.