CMSC 752 Homework 8
Morally Due Tue April 1, 2025
Dead Cat April 3

IMPORTANT In this HW when we refer to *Coloring* K_n we mean coloring the EDGES of K_n.

IMPORTANT In this HW when we ask for a table of numbers or for a number, if its a REAL number we want it to just 2 places. So for us EVERY DAY IS π-DAY, since we would use 3.14 for π.

1. (50 points) In class we proved the following two theorems.

   GP Theorem: $\forall m \ge 1$ $\forall$ 2-col of $K_{f(m)}$ $\exists$ m mono K_4's

   where $f(m) = m + 17$.

   ST Theorem: $\forall m \ge 2$ $\forall$ 2-col of $K_{g(m)}$ $\exists$ m mono K_4's

   where $g(m)$ be the least n such that

   $$n \times (n-1) \times (n-2) \times (n-3) > 73440(m-1).$$

   Note that $g(m) \sim m^{1/4}$.

   Ratio Version $\forall$ 2-col of K_n, $\exists \ge \frac{1}{3060}\binom{n}{4}$ mono K_4's. (We won't need the Ratio Version for this problem, but we will have an analog of it in Problem 2, and it will be discussed in the Extra Credit Problem.)

   (a) Make a table with five columns: m, $f(m)$, $g(m)$, $m^{1/4}$, $g(m)/m^{1/4}$, for $m = 2$ to $m = 100$. It should look like this (the numbers are fake and I only go out two rows).

m	$f(m)$	$g(m)$	$m^{1/4}$	$g(m)/m^{1/4}$
2	19	22	1.18	18.64
3	20	24	2.21	10.86

   (b) What is the least m such that $g(m) < f(m)$.

   (c) Make the table up to 1000 (do not hand that in). Find a constant A such that $g(m) = Am^{1/4}$ is a good approximation for $g(m)$. (I am assuming that the last column in your table has a limit. I have not done this problem so I could be wrong.)

2. Fill in W, X, Y, Z and prove the following Theorems. W, Y, Z are numbers. X is a statement.

GP Theorem: $\forall m \ge 1$ $\forall$ 2-col of $K_{f(m)}$ $\exists$ m mono K_5's

where $f(m) = m + W$.

ST Theorem: $\forall m \ge 2$ $\forall$ 2-col of $K_{g(m)}$ $\exists$ m mono K_5's

where $g(m)$ be the least n such that X.

Note that $g(m) \sim m^Y$.

Ratio Version $\forall$ 2-col of K_n, $\exists \ge Z\binom{n}{5}$ mono K_5's.

(a) Make a table with five columns: m, $f(m)$, $g(m)$, m^Y, $g(m)/m^Y$. for $m = 2$ to $m = 100$.

(b) What is the least m such that $g(m) < f(m)$. (I have not done this problem so I do not know if $m \le 100$.)

(c) Make the table up to 1000 (do not hand that in). Find a constant A such that $g(m) = Am^Y$ is a good approximation for $g(m)$. (I am assuming that the last column in your table has a limit. I have not done this problem so I could be wrong.)

3. (0 Points, Extra Credit)

   (a) Type your name here. (This will not get you any extra credit)

   (b) In class we proved
   Ratio Version $\forall$ 2-col of K_n $\exists \geq \frac{1}{3060}\binom{n}{4}$ mono K_4's.

   Improve the constant from $\frac{1}{3060}$ to something larger. (This might be open.)