**CMSC 752 Homework 7**
**Morally Due Tue March 25, 2025**
**Dead Cat March 27**

1. (50 points) In this problem we guide you through another proof of the Happy ending theorem.

   *For all $k \ge 3$ there exists an $n$ such that for set $X$ of $n$ points in the plane there exists $Y \subseteq X$ with $|Y| = k$ and the points in $Y$ are the vertices of a convex hull of size $k$.*

   We show that $n = R_3(k)$ suffices (for all COL: $\binom{[n]}{3} \rightarrow [2]$ there is a homog set of size $k$).

   Let $X = \{p_1, \ldots, p_n\}$.

   Let COL: $\binom{[n]}{3}$ be defined as follows:

   COL$(i < j < k) =$

   - RED if $p_i$-$p_j$-$p_k$ is CLOCKWISE
   - BLUE if $p_i$-$p_j$-$p_k$ is COUNTER-CLOCKWISE

   (Do you young folk even know what CLOCKWISE means, with your fancy digital watches?)

   FINISH THE PROOF.

2. (50 points) READ the statement of the 3-ary Can Ramsey theorem

   `https://www.cs.umd.edu/~gasarch/COURSES/752/S25/slides/aarycanramsey.pdf`

   USE the 3-ary Can Ramsey Theorem to prove the following

   *Let $X$ be a countably infinite set of points in $\mathsf{R}^2$ with no three points colinear. Show that there is an infinite $Y \subseteq X$ such that every 3-set of $Y$ encloses a different area.*

3. (0 points- Extra Credit) (In this problem all colorings are RED-BLUE colorings.)

   Proof of Disprove: Show that for all COL: $\binom{\omega^2}{2} \rightarrow [2]$ there is EITHER a RED homog $H \equiv \omega^2$ OR a BLUE homog set of size 1,000,000.

4. (0 points, DO THIS- there will be a quiz on this on Thursday. If you are NOT in class that day I will email you the quiz later.) READ the Soren Brown-William Gasarch paper on application of Ramsey Theory to history, and also the relevant parts of the papers and slides posted next to it.