**CMSC 752 Homework 5**
**Morally Due Tue March 4, 2025**
**Dead Cat March 6**

1. (50 points) Prove the following:

   **Theorem** Let $X$ be a countably infinite set of points in the plane. Then there exists an infinite $Y \subseteq X$ such that all distances between points in $Y$ are different.

   **Hint:** Use the Can Ramsey Theorem.

2. (50 points) Below are two theorems with an XXX in them (they are the same for both theorems). Fill in the XXX and prove it. (To help you with the latex I made available the latex of my slide talk on $\omega^2$ on the slides website.)

   (a) **Theorem** For all $COL\colon \binom{\omega^3}{2} \rightarrow [1,000,000]$ there is a $XXX$-homog set.

   (b) **Theorem** There exists $COL\colon \binom{\omega^3}{2} \rightarrow [XXX]$ with no $(XXX-1)$-homog set.

3. (0 points. Extra Credit.)

   Let $f$ be the function such that the following is true:

   (a) *Theorem* Let $n \ge 2$. For all $c$, for all COL: $\binom{\omega^n}{2} \rightarrow [c]$ there is an $f(n)$-homog set.

   (b) *Theorem* Let $n \ge 2$. There exists COL: $\binom{\omega^n}{2} \rightarrow [f(n)]$ with no $(f(n)-1)$-homog set.

   (a) Give an algorithm to compute $f$.

   (b) Code up the algorithm

   (c) Give a table for $f(2)$, ..., $f(20)$.

   (d) (I do not know if this is feasible as it may be two big.) Find the least $n$ such that $f(n) \ge 1,000,000$.