CMSC 752 Homework 3
Morally Due Tue Feb 18, 2025
Dead Cat Feb 20, 2025

1. (50 points) We proved in class that

   If $f : \mathsf{Z} \times \mathsf{Z} \to \mathsf{Z}$ *then there exists* $D \subseteq \mathsf{Z}$ *such that* f *restricted to* $\mathsf{D} \times \mathsf{D}$ *is NOT onto.*

   But in that proof it was quite possible for f restricted to $\mathsf{D} \times \mathsf{D}$ to just miss ONE element. So it could be ALMOST onto.

   Gary was very mad about that!

   A function f is *lazy* if there are an infinite number of elements in the co-domain such that f does not hit (so VERY not-onto).

   PROVE the following:

   If $f : \mathsf{Z} \times \mathsf{Z} \to \mathsf{Z}$ *then there exists* $D \subseteq \mathsf{Z}$ *such that* f *restricted to* $\mathsf{D} \times \mathsf{D}$ *is lazy.*

2. (50 points) Prove that, for all $f\colon \mathsf{Z} \times \mathsf{Z} \times \mathsf{Z} \to \mathsf{Z}$ there exists $\mathsf{D} \subseteq \mathsf{Z}$ such that f restricted to $\mathsf{D} \times \mathsf{D} \times \mathsf{D}$ is NOT onto.

3. (0 points, Extra Credit)

   (a) (this part gets you no credit) Give your name (this will not get you any extra credit, but since I grade this one by putting the names of who got it right into a file, this makes my life easier.)

   (b) Find some value c and prove the following:

   For all COL: $\binom{\omega^2}{2} \rightarrow [1,000,000]$ *there exists a c-homog set.*

   Some points about this.

   - Your answer had to be well written.
   - The optimal answer is $c = 4$ though I doubt you can obtain that. I am really looking for a NOT-MESSY proof of a weaker-than-known result.
   - The first step you probably all know: WITHIN each copy of ω 2-ary Ramsey, and then for all of the copies of ω use 1-ary Ramsey. So you can just ASSUME that within each copy of ω, all of the edges are RED. (No extra credit for getting just that far.)
   - Some people tried to use the Infinite Bipartite Ramsey Theorem on all pairs. This won't work since that yields 3-homog which is not true. However, if you can get something like this to work (I couldn't) that would be great.