Homework 2
Morally Due Tue Feb 11, 2025, at 3:30PM
Dead Cat Feb 13, 2025, at 3:30PM

1. (30 points)

   Recall: If L_1, L_2 are two ordered sets then $L_1 \equiv L_2$ means there is an order-preserving bijection between L_1 and L_2.

   Let 5ω be $\omega + \omega + \omega + \omega + \omega$.

   This ordering is five copies of $\omega = \{1 < 2 < 3 < \cdots\}$ concatenated.

   Fill in the value of c in the next two statements (the same value) and prove both parts. You may use the infinite Ramsey Theorem and the Infinite Bipartite Ramsey Theorm.

   - There exists COL: $\binom{5\omega}{2} \rightarrow [\mathrm{c}]$ such that there is NO $(c-1)$-homog $H \equiv 5\omega$.
   - For all COL: $\binom{5\omega}{2} \rightarrow [1,000,000]$ there exists a c-homog $H \equiv 5\omega$.

2. (30 points)

   Recall: If L_1, L_2 are two ordered sets then $L_1 \equiv L_2$ means there is an order-preserving bijection between L_1 and L_2.

   Let ω^2 be $\omega + \omega + \cdots$.

   Fill in the value of c in the next two statements (the same value) and prove both parts. You may use the infinite Ramsey Theorem and the Infinite Bipartite Ramsey Theorm.

   - There exists $\mathrm{COL}\colon \binom{\omega^2}{2} \rightarrow [\mathrm{c}]$ such that there is NO $(c-1)$-homog $H \equiv \omega^2$.
   - For all $\mathrm{COL}\colon \binom{\omega^2}{2} \rightarrow [1,000,000]$ there exists a c-homog $H \equiv \omega^2$.

3. (40 points)

   In class we proved the following:

   The Infinite Ramsey Theorem,

   The Finite Ramsey Theorem with a proof that gave no bounds on $R(k)$

   The Finite Ramsey Theorem with a proof that gave the bound $R(k) \le 2^{2k-1}$ This last proof followed the proof of the *Infinite Ramsey Theorem* closely.

   We then proved

   Infinite 3-hypergraph Ramsey Theorem

   YOUR ASSIGNMENT:

   Prove the Finite 3-hypergraph Ramsey Theorem with a proof that gave a bound on $R(k)$.

   This prove should follow the proof of the *Infinite 3-hypergraph Ramsey Theorem* closely.

   Make sure to state what the upper bound you get is.

   Hint The proof of the 2-ary Ramsey Theorem used the 1-ary Ramsey Theorem. You will need the following restatement of 2-ary

   For all $\mathrm{COL}\colon \binom{[n]}{2} \to [2]$ *there exists a homog set of size* $\ge 0.5 \log n$.