HW 01 CMSC 752

Morally Due Feb 4 3:30PM. Dead-Cat Due Feb 6 at 3:30PM

Course Website

`https://www.cs.umd.edu/~gasarch/COURSES/752/S25/index.html`

1. (0 points but do it anyway.)

   (a) If you are not on piazza, put yourself on piazza.

   (b) If you are not on gradescope, put yourself on gradescope. The Entry Code is DKW6VX. (The school automatically put everyone enrolled in the class on gradescope but it might have messed up, and some of you might not be on the roster yet.)

   (c) (0 points) Learn LaTeX and write some simple documents in it.

   (d) (0 points) Learn the package tikz that you can use to draw pictures in LaTeX.

2. (50 points) *Let* $p_1, p_2, \ldots$ *be an infinite set of distinct reals. Show that at least one of the following is true:*

   - *There exists an infinite increasing subsequence (so there exists* $i_1 < i_2 < \cdots$ *such that* $p_{i_1} < p_{i_2} < \cdots$*.)*
   - *There exists an infinite decreasing subsequence (so* $i_1 < i_2 < \cdots$ *such that* $p_{i_1} > p_{i_2} > \cdots$*.)*

   Hint: Consider the coloring COL: $\binom{\mathsf{N}}{2} \rightarrow \{INC, DEC\}$ defined by (assume $i < j$)

$$\mathrm{COL}(i,j) = \begin{cases} INC & \text{if } p_i < p_j \\ DEC & \text{if } p_i > p_j \end{cases} \tag{1}$$

3. (a) (0 points but do it) The theorem in Part 1 is a Lemma in the proof of the Bolzano-Weierstrass theorem. See the proof on Wikipedia

   `https://en.wikipedia.org/wiki/Bolzano%E2%80%93Weierstrass_theorem`

   Look at the proof on Wikipedia. Is it the same proof as you gave in Part 1, only messier? Or is it a different proof?

   (b) (0 points but DO IT)

   Listen to The Bolzano Weierstrass Rap:

   `https://www.youtube.com/watch?v=dfO18klwKHg`

   Then list to any three other RANDOM math songs from my collection, which is here:

   `https://www.cs.umd.edu/~gasarch/FUN/mathsongs.html`

   Which was your favorite song?

   Which was your least favorite song?

   Was your least favorite song better than the BW rap

4. (50 points) (This is not a question in Ramsey Theory but it is a prelude to one of our "Applications.")

   Notation Z is the integers. $\mathsf{Z}[x_1, \ldots, x_n]$ is the set of all polynomials in $\{x_1, \ldots, x_n\}$ (they do not have to all appear) with coefficients in Z.

   For $p(x, y) \in \mathsf{Z}[x, y]$, we view p as a function from $Z \times \mathsf{Z}$ into Z.

   Note that $p(x, y)$ might be onto (e.g., $p(x, y) = x + y + 1$) or not (e.g., $p(x, y) = x^2 + y^2 + 1$).

   Prove the following;

   FOR ALL $p(x, y) \in Z[x, y]$ there is an infinite set $\mathsf{D} \subseteq \mathsf{Z}$ such that p restricted to $\mathsf{D} \times \mathsf{D}$ is NOT onto Z (that is, there is some element of Z not in the image).

5. (EXTRA CREDIT) Show that FOR ALL $p(x,y) \in Z[x,y]$, if we let $q(x,y) = \lceil p(x,y)^{1/101} \rceil$ (Note that a number has many 101th roots, but only one real one. Take the real one.) then there is an infinite set $\mathsf{D} \subseteq \mathsf{Z}$ such that q restricted to $\mathsf{D} \times \mathsf{D}$ is NOT onto Z (that is, there is some element of Z not in the image).

6. (EXTRA CREDIT) Prove or Disprove: FOR ALL function $p :: \mathsf{Z} \times \mathsf{Z} \rightarrow \mathsf{Z}$ there is an infinite set $\mathsf{D} \subseteq \mathsf{Z}$ such that p restricted to $\mathsf{D} \times \mathsf{D}$ is NOT onto Z (that is, there is some element of Z not in the image).