BILL START RECORDING

HW 09 Solutions

Problem 1

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We assume $\Sigma = \{a, b\}$.

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We do this problem without using O-of. We will assume t is large.

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$A \rightarrow e$ Number of rules of this type: t.

Number of rules: $t^5 + t^4 + t^3 + 2t + t \leq 2t^5$.

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Hence there are $\leq 2^{2t^5}$ grammars.

We let $g(t) = 2^{2t^5}$.

Problem 1c

Find a function h such that the following is true:
$\exists \boldsymbol{w} \in \{\boldsymbol{a}, \boldsymbol{b}\}^*$ of length $\leq h(t)$ such that there is NO Leo-Grammar G with t nonterminals such that $L(G) = \{w\}$.

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Let $h(t) = 3t^5$.

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Let $h(t) = 3t^5$.
There are 2^{3t^5} strings of length $3t^5$.

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There are $\leq 2^{2t^5}$ Grammars.
Let $h(t) = 3t^5$.
There are 2^{3t^5} strings of length $3t^5$.

Hence there must be some w of length $3t^5$ that is not generated by ANY Leo grammar with t NTs.

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$CA \rightarrow AC$

$BC \rightarrow CB$

$CB \rightarrow BC$

$A \rightarrow a$

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$A \to a$
$B \to b$

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Number of rules: $2 + 6 + 3 = 11$.

Problem 2b

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Give a CSG for the language

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$S \to SA_1A_2A_3 \cdots A_n \mid e$
(for future: 2 rules)

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For all $\leq i < j \leq n$
$A_iA_j \to A_jA_i$.
$A_jA_i \to A_iA_n$.

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For all $\leq i < j \leq n$
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(for future: $2\binom{n}{2} = n(n-1)$ rules)

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For all $1 \leq i \leq n$
$A_i \to a_i$

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$A_jA_i \to A_iA_n$.
(for future: $2\binom{n}{2} = n(n-1)$ rules)

For all $1 \leq i \leq n$
$A_i \to a_i$
(for future: n rules.)

Problem 2c

Find a function r such that your CSG from the last part has $\leq r(n)$ rules.

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Let $r(n) = n^2 + 2$.