DFA Classifiers for Mods

Pattern for Mod 14

$$10^0 \equiv 1$$

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$10^0 \equiv 1$

$10^1 \equiv 10$
$10^2 \equiv 10 \times 10 \equiv 2$
$10^3 \equiv 2 \times 10 \equiv 6$
$10^4 \equiv 6 \times 10 \equiv 4$
$10^5 \equiv 4 \times 10 \equiv 12$
$10^6 \equiv 12 \times 10 \equiv 8$
$10^7 \equiv 8 \times 10 \equiv 10$

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$$10^7 \equiv 8 \times 10 \equiv 10$$
$$(1, \overline{10, 2, 6, 4, 12, 8}).$$

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AFTER start state need to keep track of

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$(1, \overline{10, 2, 6, 4, 12, 8})$.
AFTER start state need to keep track of
(1) weighted sum mod 14 and (2) digit position mod 6.

DFA for Mod 14

Idea Have a start state that goes into a grid-DFA for mod 14.

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$\delta((q,4),\sigma) = (q + 12 \times \sigma \pmod{14}, 5)$

$\delta((q,5),\sigma) = (q + 8 \times \sigma \pmod{14}, 0)$

DFA for Mod n, $(a_1, \overline{b_0, \ldots, b_{m-1}})$

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$\delta((q,0),\sigma) = (q + b_0 \times \sigma \pmod{n}, 1)$

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$$\delta((q,0),\sigma) = (q + b_0 \times \sigma \pmod{n}, 1)$$
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$\delta((q,m-1),\sigma) = (q + b_{m-1} \times \sigma \pmod{n}, 0)$

Mod 15

$$10^0 \equiv 1$$

Mod 15

$10^0 \equiv 1$

$10^1 \equiv 10$
$10^2 \equiv 100 \equiv 10$
$(1, \overline{10})$.

Mod 15

$10^0 \equiv 1$

$10^1 \equiv 10$
$10^2 \equiv 100 \equiv 10$
$(1, \overline{10})$.

Use formula: $1 + 15 \times 1 = 16$.

Pattern for Mod 16

$10^0 \equiv 1$
$10^1 \equiv 10$
$10^2 \equiv 4$
$10^3 \equiv 8$
$10^4 \equiv 0$ ALL REST ARE 0.

Pattern for Mod 16

$10^0 \equiv 1$
$10^1 \equiv 10$
$10^2 \equiv 4$
$10^3 \equiv 8$
$10^4 \equiv 0$ ALL REST ARE 0.

$(1, 10, 4, 8, \overline{0})$.

DFA for Mod 16

DFA for Mod 16

Weights $(1, 10, 4, 8, \overline{0})$.

DFA for Mod 16

Weights $(1, 10, 4, 8, \overline{0})$.

$Q = \{s\} \cup \{(q, j) \colon 0 \le i \le 15 \quad \wedge \quad 0 \le j \le 3\}$

DFA for Mod 16

Weights $(1, 10, 4, 8, \overline{0})$.

$Q = \{s\} \cup \{(q, j) \colon 0 \leq i \leq 15 \quad \wedge \quad 0 \leq j \leq 3\}$
$|Q| = 1 + 16 \times 4 = 65$.

DFA for Mod 16

Weights $(1, 10, 4, 8, \overline{0})$.

$Q = \{s\} \cup \{(q, j) \colon 0 \leq i \leq 15 \quad \wedge \quad 0 \leq j \leq 3\}$
$|Q| = 1 + 16 \times 4 = 65.$

For all $0 \leq \sigma \leq 9$, $\delta(s, \sigma) = (1 \times \sigma, 0)$.

DFA for Mod 16

Weights $(1, 10, 4, 8, \overline{0})$.

$Q = \{s\} \cup \{(q, j) \colon 0 \leq i \leq 15 \quad \wedge \quad 0 \leq j \leq 3\}$
$|Q| = 1 + 16 \times 4 = 65.$

For all $0 \leq \sigma \leq 9$, $\delta(s, \sigma) = (1 \times \sigma, 0)$.

For all $0 \leq \sigma \leq 9$, $\delta((q, 0), \sigma) = (q + 10 \times \sigma \pmod{16}, 1)$

DFA for Mod 16

Weights $(1, 10, 4, 8, \overline{0})$.

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For all $0 \leq \sigma \leq 9$, $\delta((q, 0), \sigma) = (q + 10 \times \sigma \pmod{16}, 1)$
For all $0 \leq \sigma \leq 9$, $\delta((q, 1), \sigma) = (q + 4 \times \sigma \pmod{16}, 2)$

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Weights $(1, 10, 4, 8, \overline{0})$.

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For all $0 \leq \sigma \leq 9$, $\delta((q, 2), \sigma) = (q + 8 \times \sigma \pmod{16}, 3)$

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For all $0 \leq \sigma \leq 9$, $\delta((q, 3), \sigma) = (q, 3)$.

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Weights $(1, 10, 4, 8, \overline{0})$.

$Q = \{s\} \cup \{(q, j) \colon 0 \le i \le 15 \quad \wedge \quad 0 \le j \le 3\}$
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For all $0 \le \sigma \le 9$, $\delta((q, 3), \sigma) = (q, 3)$.
Can we do this with < 65 states?

Can Do With Slightly Less Than 65 States

$$Q = \{s\} \cup \{(q, j) \colon 0 \leq i \leq 15 \quad \wedge \quad 0 \leq j \leq 3\}$$

Can Do With Slightly Less Than 65 States

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Once the machine leave state $(0, 0)$, …, $(15, 0)$ will never return.

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Once the machine leave state $(0, 0)$, . . ., $(15, 0)$ will never return.

Key Will never use $(10, 0)$, $(11, 0)$, $(12, 0)$, $(13, 0)$, $(14, 0)$, $(15, 0)$.

Can Do With Slightly Less Than 65 States

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So only need $65 - 6 = 59$.

Can Do With Slightly Less Than 65 States

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Once the machine leave state $(0,0)$, ..., $(15,0)$ will never return.

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So only need $65 - 6 = 59$.

An answer of 65 still gets full credit.

Can Do With Slightly Less Than 65 States

$Q = \{s\} \cup \{(q, j) \colon 0 \le i \le 15 \quad \wedge \quad 0 \le j \le 3\}$
$|Q| = 1 + 16 \times 4 = 65.$

For all $0 \le \sigma \le 9$, $\delta(s, \sigma) = (1 \times \sigma, 0)$.

Once the machine leave state $(0, 0)$, ..., $(15, 0)$ will never return.

Key Will never use $(10, 0)$, $(11, 0)$, $(12, 0)$, $(13, 0)$, $(14, 0)$, $(15, 0)$.

So only need $65 - 6 = 59$.

An answer of 65 still gets full credit.

Can you do better than 59 states? I leave this to you.

Pattern for Mod 17

After calcluation the pattern is

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$\overline{1, 10, 15, 14, 4, 6, 9, 5, 16, 7, 2, 3, 13, 11, 8, 12}$

Pattern for Mod 17

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$\overline{1, 10, 15, 14, 4, 6, 9, 5, 16, 7, 2, 3, 13, 11, 8, 12}$

Mod is 17, pattern is length 16.

Pattern for Mod 17

After calcluation the pattern is

$\overline{1, 10, 15, 14, 4, 6, 9, 5, 16, 7, 2, 3, 13, 11, 8, 12}$

Mod is 17, pattern is length 16.

DFA size $17 \times 16 = 272$.

Pattern for Mod 18

Pattern for Mod 18

$$10^0 \equiv 1$$
$$10^1 \equiv 10$$
$$10^2 \equiv 10$$

DFA for Mod 18

DFA for Mod 18

Weights $(1, \overline{10})$.

DFA for Mod 18

Weights $(1, \overline{10})$.

$Q = \{s\} \cup \{i\colon 0 \le i \le 17\}$

DFA for Mod 18

Weights $(1, \overline{10})$.

$Q = \{s\} \cup \{i : 0 \leq i \leq 17\}$
$|Q| = 1 + 18 = 19.$
Weights $(1, \overline{10})$.

DFA for Mod 18

Weights $(1, \overline{10})$.

$Q = \{s\} \cup \{i : 0 \le i \le 17\}$
$|Q| = 1 + 18 = 19.$
Weights $(1, \overline{10})$.

For all $0 \le \sigma \le 9$, $\delta(s, \sigma) = 1 \times \sigma$

DFA for Mod 18

Weights $(1, \overline{10})$.

$Q = \{s\} \cup \{i \colon 0 \leq i \leq 17\}$
$|Q| = 1 + 18 = 19$.
Weights $(1, \overline{10})$.

For all $0 \leq \sigma \leq 9$, $\delta(s, \sigma) = 1 \times \sigma$

For all $0 \leq \sigma \leq 9$, $\delta(q, \sigma) = q + 10 \times \sigma \pmod{18}$

This Pattern and DFA May Be on Future HW

Let $n \geq 10$.

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If pattern is $(a_0, \ldots, a_{L-1}, \overline{b_0, \ldots, b_m})$

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Let $n \ge 10$.

If pattern is $(a_0, \ldots, a_{L-1}, \overline{b_0, \ldots, b_m})$

then DFA is of size XXX.

This Pattern and DFA May Be on Future HW

Let $n \geq 10$.

If pattern is $(a_0, \ldots, a_{L-1}, \overline{b_0, \ldots, b_m})$

then DFA is of size XXX.

This Question may be on a future HW so work on while you are still in the mindset.

This Pattern and DFA May Be on Future HW

Let $n \ge 10$.

If pattern is $(a_0, \ldots, a_{L-1}, \overline{b_0, \ldots, b_m})$

then DFA is of size XXX.

This Question may be on a future HW so work on while you are still in the mindset.

There are several cases.

PROBLEM 2

PROBLEM 2

a) DFA for $L_1 = \{a^n \colon n \equiv 0 \pmod{6}\}$

PROBLEM 2

a) DFA for $L_1 = \{a^n \colon n \equiv 0 \pmod{6}\}$

$Q = \{0, 1, 2, 3, 4, 5\}$

PROBLEM 2

a) DFA for $L_1 = \{a^n : n \equiv 0 \pmod 6\}$

$Q = \{0, 1, 2, 3, 4, 5\} \qquad s = 0$

PROBLEM 2

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e) Thought question: What about $L_1 \cup L_2$? DISCUSS.

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3) Discuss: Do there exists L_1, L_2 such that DFA **requires** $n_1 n_2$?

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This is why DFAs are so useful in practice.