Number of States for DFAs and NFAs

Consider DFAs For $\{aaa\}$

$\Sigma = \{a\}$.

Consider DFAs For {aaa}

$\Sigma = \{a\}$.

$$L = \{aaa\}$$

Consider DFAs For $\{aaa\}$

$\Sigma = \{a\}$.

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With your table draw a DFA for this language.

Consider DFAs For $\{aaa\}$

$\Sigma = \{a\}$.

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With your table draw a DFA for this language.

How many states does it have?

Consider DFAs For {aaa}

$\Sigma = \{a\}$.

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With your table draw a DFA for this language.

How many states does it have?

5

Consider DFAs For $\{aaa\}$

$\Sigma = \{a\}$.

$$L = \{aaa\}$$

With your table draw a DFA for this language.

How many states does it have?

5

Is there a DFA for L that has 4 states? Discuss.

Any DFA for $\{aaa\}$ Has ≥ 5 states

Assume there exists a DFA M for $\{aaa\}$ with 4 states.

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Input aaa

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Starts in state $s = q_0$

Any DFA for $\{aaa\}$ Has ≥ 5 states

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Input aaa

Starts in state $s = q_0$
an a is processed.

Any DFA for $\{aaa\}$ Has ≥ 5 states

Assume there exists a DFA M for $\{aaa\}$ with 4 states.
Input aaa

Starts in state $s = q_0$
an a is processed. Now in state q_1. $q_1 \notin F$.

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Assume there exists a DFA M for $\{aaa\}$ with 4 states.
Input aaa

Starts in state $s = q_0$
an a is processed. Now in state q_1. $q_1 \notin F$.
another a is processed.

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Assume there exists a DFA M for $\{aaa\}$ with 4 states.
Input aaa

Starts in state $s = q_0$
an a is processed. Now in state q_1. $q_1 \notin F$.
another a is processed. Now in state q_2. $q_2 \notin F$.

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Starts in state $s = q_0$
an a is processed. Now in state q_1. $q_1 \notin F$.
another a is processed. Now in state q_2. $q_2 \notin F$.
another a is processed.

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Starts in state $s = q_0$
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another a is processed. Now in state q_2. $q_2 \notin F$.
another a is processed. Now in state q_3. $q_3 \in F$.

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Input aaa

Starts in state $s = q_0$
an a is processed. Now in state q_1. $q_1 \notin F$.
another a is processed. Now in state q_2. $q_2 \notin F$.
another a is processed. Now in state q_3. $q_3 \in F$.

$\delta(q_3, a)$ has to be one of q_0, q_1, q_2, q_3.

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Input aaa

Starts in state $s = q_0$
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another a is processed. Now in state q_2. $q_2 \notin F$.
another a is processed. Now in state q_3. $q_3 \in F$.

$\delta(q_3, a)$ has to be one of q_0, q_1, q_2, q_3.
Let say its q_1 (the other cases are similar).

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Assume there exists a DFA M for $\{aaa\}$ with 4 states.
Input aaa

Starts in state $s = q_0$
an a is processed. Now in state q_1. $q_1 \notin F$.
another a is processed. Now in state q_2. $q_2 \notin F$.
another a is processed. Now in state q_3. $q_3 \in F$.

$\delta(q_3, a)$ has to be one of q_0, q_1, q_2, q_3.
Let say its q_1 (the other cases are similar).
aaa ends in state q_3.

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another a is processed. Now in state q_3. $q_3 \in F$.

$\delta(q_3, a)$ has to be one of q_0, q_1, q_2, q_3.
Let say its q_1 (the other cases are similar).
aaa ends in state q_3.
$aaaa$ ends in state q_1.

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Assume there exists a DFA M for $\{aaa\}$ with 4 states.
Input aaa

Starts in state $s = q_0$
an a is processed. Now in state q_1. $q_1 \notin F$.
another a is processed. Now in state q_2. $q_2 \notin F$.
another a is processed. Now in state q_3. $q_3 \in F$.

$\delta(q_3, a)$ has to be one of q_0, q_1, q_2, q_3.
Let say its q_1 (the other cases are similar).
aaa ends in state q_3.
$aaaa$ ends in state q_1.
$aaaaa$ ends in state q_2.

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Input aaa

Starts in state $s = q_0$
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another a is processed. Now in state q_3. $q_3 \in F$.

$\delta(q_3, a)$ has to be one of q_0, q_1, q_2, q_3.
Let say its q_1 (the other cases are similar).
aaa ends in state q_3.
$aaaa$ ends in state q_1.
$aaaaa$ ends in state q_2.
$aaaaaa$ ends in state q_3, so $aaaaaa$ is accepted. Contradiction.

What about $\{a^n\}$?

More generally: For all n

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There is a DFA for $\{a^n\}$ with $n + 2$ states.

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More generally: For all n

There is a DFA for $\{a^n\}$ with $n + 2$ states.

Any DFA for $\{a^n\}$ has $\geq n + 2$ states.

Consider NFAs For {aaa}

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How many states does it have?

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Consider NFAs For $\{aaa\}$

$\Sigma = \{a\}$.

$$L = \{aaa\}$$

With your table draw an NFA for this language.

How many states does it have?

4

Is there an NFA for L that has 3 states? Discuss.

Any NFA for $\{aaa\}$ Has $\geq$ 4 states

Assume there exists a NFA M for $\{aaa\}$ with 4 states.

Any NFA for $\{aaa\}$ Has $\geq$ 4 states

Assume there exists a NFA M for $\{aaa\}$ with 4 states.
Input aaa. We look at the ACCEPTING path.

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Starts in state $s = q_0$

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Input aaa. We look at the ACCEPTING path.

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an a is processed.

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Input aaa. We look at the ACCEPTING path.

Starts in state $s = q_0$
an a is processed. Now in state q_1. $q_1 \notin F$.

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Input aaa. We look at the ACCEPTING path.

Starts in state $s = q_0$
an a is processed. Now in state q_1. $q_1 \notin F$.
another a is processed.

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Input aaa. We look at the ACCEPTING path.

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an a is processed. Now in state q_1. $q_1 \notin F$.
another a is processed. Now in state q_2. $q_2 \notin F$.

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another a is processed. Now in state q_3. $q_3 \in F$.

Look at q_0, q_1, q_2, q_3.

Any NFA for $\{aaa\}$ Has ≥ 4 states

Assume there exists a NFA M for $\{aaa\}$ with 4 states.
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another a is processed. Now in state q_3. $q_3 \in F$.

Look at q_0, q_1, q_2, q_3. Two of them have to be the same.

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an a is processed. Now in state q_1. $q_1 \notin F$.
another a is processed. Now in state q_2. $q_2 \notin F$.
another a is processed. Now in state q_3. $q_3 \in F$.

Look at q_0, q_1, q_2, q_3. Two of them have to be the same.

Can use this to find a shorter string that is accepted.
Contradiction.

DFAs and NFAs for $\{a^n\}$?

For all n

DFAs and NFAs for $\{a^n\}$?

For all n

There is a DFA for $\{a^n\}$ with $n + 2$ states.

DFAs and NFAs for $\{a^n\}$?

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Any DFA for $\{a^n\}$ has $\geq n+2$ states.

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There is an NFA for $\{a^n\}$ with $n+1$ states.

DFAs and NFAs for $\{a^n\}$?

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There is a DFA for $\{a^n\}$ with $n+2$ states.

Any DFA for $\{a^n\}$ has $\geq n+2$ states.

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Any NFA for $\{a^n\}$ has $\geq n+1$ states.

DFA for $\{a^i : i \neq 1000\}$

$L = \{a^i : i \neq 1000\}$

DFA for $\{a^i : i \neq 1000\}$

$L = \{a^i : i \neq 1000\}$
There is a DFA for this with 1002 states.

DFA for $\{a^i : i \neq 1000\}$

$L = \{a^i : i \neq 1000\}$
There is a DFA for this with 1002 states.

Is there a DFA with ≤ 1001 states.

DFA for $\{a^i : i \neq 1000\}$

$L = \{a^i : i \neq 1000\}$
There is a DFA for this with 1002 states.

Is there a DFA with ≤ 1001 states.

No.

DFA for $\{a^i : i \neq 1000\}$

$L = \{a^i : i \neq 1000\}$
There is a DFA for this with 1002 states.

Is there a DFA with ≤ 1001 states.

No.

If there was, then complement it to get
a DFA for $\{a^{1000}\}$ with ≤ 1001 states.

DFA for $\{a^i : i \neq 1000\}$

$L = \{a^i : i \neq 1000\}$
There is a DFA for this with 1002 states.

Is there a DFA with ≤ 1001 states.

No.

If there was, then complement it to get
a DFA for $\{a^{1000}\}$ with ≤ 1001 states.

Contradiction.

NFA for $\{a^i : i \neq 1000\}$

There is an NFA for L that has 1001 states.

NFA for $\{a^i : i \neq 1000\}$

There is an NFA for L that has 1001 states.
Work in groups to see if you can do better, and not just be a few. For definitness: Can you get an NFA with ≤ 900 states?

NFA for $\{a^i : i \neq 1000\}$

There is an NFA for L that has 1001 states.
Work in groups to see if you can do better, and not just be a few. For definitness: Can you get an NFA with ≤ 900 states?
VOTE

NFA for $\{a^i : i \neq 1000\}$

There is an NFA for L that has 1001 states.
Work in groups to see if you can do better, and not just be a few. For definitness: Can you get an NFA with ≤ 900 states?
VOTE

1. There is an NFA for L with ≤ 900 states.

NFA for $\{a^i : i \neq 1000\}$

There is an NFA for L that has 1001 states.
Work in groups to see if you can do better, and not just be a few. For definitness: Can you get an NFA with ≤ 900 states?
VOTE

1. There is an NFA for L with ≤ 900 states.
2. All NFA's for L have ~ 1000 states.

Much Less Than 1000 States

There is a an NFA for

$$\{a^i : i \neq 1000\}$$

with MUCH LESS than 1000 states.
That will be the next lecture.

This Slide Packet / Future Slide Packets

This slide packet had langs where NFAs were not much more powerful then DFA.

This Slide Packet / Future Slide Packets

This slide packet had langs where NFAs were not much more powerful then DFA.

We now go to a slide packet where NFAs do MUCH better than DFAs.