HW 06 CMSC 452
Morally Due TUES March 11 11:00AM
Dead-Cat Due THU March 13 at 11:00AM

1. (30 points) Give a CFG for the following languages
   The alphabet is $\{a, b\}$.

   (a) $\{w : \#_a(w) \equiv 0 \pmod 3\}$.
   (b) $\{w : \#_a(w) \equiv 1 \pmod 3\}$.

2. (30 points) Give a context free grammar for the following language $\overline{\{a^n b^n c^n : n \in \mathbb{N}\}}$.

3. (40 points)

   (a) (20 points) Give a function f such that the following is true, and prove it:

   If α is a regex of length n then there exists a context free grammar G such that $L(G) = L(\alpha)$ with $f(n)$ rules ($f(n)$ may use O-notation).

   (The CFG DOES NOT have to be in Chomsky Normal Form.)

   (Your proof should be an algorithm that takes n and produces the grammar. You may use any theorem in class as a subroutine for this algorithm.)

   (b) (20 points) Give a function g such that the following is true, and prove it:

   The language

$$L_n = \{w \colon \#_a(w) \equiv 0 \pmod{n} \wedge \#_b(w) \equiv 0 \pmod{n}\}$$

   has a context free grammar with $g(n)$ rules ($g(n)$ may use O-notation).

   (The CFG DOES NOT have to be in Chomsky Normal Form.)

   (Your proof should be an algorithm that takes n and produces the grammar. You may use any theorem in class as a subroutine for this algorithm.)