HW 05 CMSC 452
Morally Due TUES March 4 11:00AM
Dead-Cat Due THU March 6 at 11:00AM

1. For this problem

   - The *size* of a DFA is the number of states.
   - The *size* of an NFA is the number of states.
   - The *size* of a regex is its length.

   This is just a FILL IN THE BLANK. You may use O-notation. I give two examples.

   EXAMPLE ONE

   If the question was:

   There is an algorithm that will, given two regex's α_1, α_2 of sizes n_1, n_2, returns a regex for $L(\alpha_1)L(\alpha_2)$ of size FILLIN.

   The answer would be $n_1 + n_2 + O(1)$.

   EXAMPLE TWO

   There is an algorithm that will, given a DFA M of size n, returns a regex for $L(M)$ of size FILLIN.

   the answer is $2^{O(n)}$. (This will be one of the questions below and I've just given you the answer. Yeah for you!)

   The ACTUAL QUESTIONS are on the next page

(a) There is an algorithm that will, given two DFA's M_1, M_2 of sizes n_1, n_2, returns a DFA for $L(M_1) \cap L(M_2)$ of size FILLIN.

(b) There is an algorithm that will, given two DFA's M_1, M_2 of sizes n_1, n_2, returns an NFA for $L(M_1) \cdot L(M_2)$ of size FILLIN.

(c) There is an algorithm that will, given a regex α of length n, returns an NFA for $L(\alpha)$ of size FILLIN.

(d) There is an algorithm that will, given a DFA M of size n, returns a regex for $L(M)$ of size FILLIN.

2. PROVE the following statements by giving an algorithm (your algorithm may use the algorithms in problem 1 as subroutines) and fill in where it says FILLIN. No proof of the FILLIN is needed.

   (a) There is an algorithm that will, given two DFA's M_1, M_2 of sizes n_1, n_2, returns a DFA for $L(M_1) \cdot L(M_2)$ of size FILLIN. (NOTE: in Problem 1 we asked for going from two DFA's to an NFA. Here we are asking to go from two DFA's to a DFA.)

   (b) There is an algorithm that will, given two regex's α_1, α_2 of sizes n_1, n_2, returns a regex for $L(\alpha_1) \cap L(\alpha_2)$ of size FILLIN.

3. For each of the following state if its is REGULAR or NOT REGULAR.
   If you say REGULAR then give either a DFA or REGEX for it.
   If you say NOT REGULAR than prove it using the pumping lemma.

   (a) $\{a^{\lfloor \log_2(n) \rfloor} : n \geq 1\}$
   (b) $\{a^{2n} : n \geq 1\}$
   (c) $\{a^{2^n} : n \geq 1\}$