1. (30 points) Let L be regular via DFA $(Q, \Sigma, \delta, s, F)$.

   Write down an NFA $(Q', \Sigma, \delta', s', F')$ for L^*.

2. (30 points) In this problem $\Sigma = \{a, b, c\}$. Let L be the set of all w such that the following hold:

- $\#_a(w) \equiv 1 \pmod{3}$, AND
- $\#_b(w) \equiv 2 \pmod{4}$, AND
- $\#_c(w) \equiv 3 \pmod{5}$}.

Write a DFA for L in table form. Give Q, δ, s, F. (We already know Σ.)

3. (40 points) Give an NFA for

$$L = \{a^i : i \neq 100\}.$$

that has substantially less than 100 states.

A few points about this:

- You should use the technique on the SMALL NFA slides. (The next two points make that point.)

- You NEED to use the Chicken McNugget Theorem. You may use it without proof.

- You NEED to use the following Theorem.

  Let $n \in \mathbb{N}$. Let $p_1, \ldots, p_m$ be primes such that $p_1 \cdots p_m \geq m$. Let $i \in \mathbb{N}$. Let X be the set of all i such that

  $i \not\equiv n \pmod{p_1}$

  $i \not\equiv n \pmod{p_2}$

  $\vdots \qquad \vdots$

  $i \not\equiv n \pmod{p_m}$

  Then (1) X contains $\{0, 1, \ldots, n-1\}$, (2) X does not contain n, and (3) it will contain some elements $> n$ but we don't care.

- You can give either a NEATLY DRAWN diagram or a table.

- DO NOT optimize. Lets say you get it to be 50 states. DO NOT try to make it 49.