HW 02 CMSC 452
Morally Due TUES Feb 11 11:00AM
Dead-Cat Due THU Feb 13 at 11:00AM

1. (50 points) For this problem you might want to write a program that will, given n, compute $10^0 \pmod n$, $10^1 \pmod n$, $10^2 \pmod n$ etc and find a pattern. If you write such a program KEEP IT - you may need it later in the course.

   Write a nice clean easy-to-grade table for the answer. I give an example later.

   $\overline{a}$ means $(a, a, \ldots)$.

   $\overline{a, b}$ means $(a, b, a, b, \ldots)$.

   For $n = 14, 15, 16, 17, 18, 19, 20$ give the following

   (a) The weights needed for a DFA classifier for mod n. E.g.,
   If $n = 2$ the answer is $(1, \overline{0})$
   If $n = 3$ the answer is $(\overline{1})$
   If $n = 4$ the answer is $(1, 2, \overline{0})$
   If $n = 11$ the answer is $(\overline{1, -1})$

   (b) The size of a DFA classifier for determining what a number is congruent to mod n.
   (For this problem, UNLIKE CLASS, we will count the start state as having input 0 so $\equiv 0 \pmod n$. E.g.,
   If $n = 2$ the answer is 3.
   If $n = 3$ the answer is 3.
   If $n = 4$ I leave that to you to figure out.
   If $n = 11$ the answer is 22.

   The format for the answer should be like this:

n	pattern	Numb of States
2	$(1, \overline{0})$	3
3	$(\overline{1})$	3
4	$(1, 2, \overline{0})$	18 NOT the real answer
11	$(\overline{1, -1})$	22

2. (50 points) In this problem $\Sigma = \{a\}$.

   (a) (10 points) Write a DFA for

   $$L_1 = \{a^n : n \equiv 0 \pmod 6\}.$$

   How many states does it have?

   (b) (10 points) Write a DFA for

   $$L_2 = \{a^n : n \equiv 0 \pmod 9\}.$$

   How many states does it have?

   (c) (10 points) IF you were to use the construction to get a DFA for $L_1 \cap L_2$ how many states would it have? (DO NOT do the construction.)

   (d) (20 points) Give a DFA for $L_1 \cap L_2$ that uses fewer states then the one from the construction.

   (e) (THOUGHT QUESTION- DO NOT SUBMIT) IF you were to use the construction to get a DFA for $L_1 \cup L_2$ how many states would it have? (DO NOT do the construction.) IS THERE a DFA for $L_1 \cup L_2$ with fewer states than the one from construction?