

HW 1 CMSC 452. Morally DUE Feb 6

NOTE- IN PROBLEMS 2 and 3 YOU ARE ASKED TO PROVE THEOREMS YOU MAY USE THESE THEOREMS IN PROBLEM 4

1. (0 points) What is your name? Write it clearly. Staple your HW. When is the midterm? Where is the midterm?

AN INJECTION IS ALSO CALLED A 1-1 MAPPING.

2. (25 points) Prove that if there is an injection from A to B and an injection from B to A then there is a bijection from A to B (this is called the Cantor-Schroder-Bernstein by some and the Schroder-Bernstein theorem by others, and likely other combinations by other people. You MAY go to the web and find a proof; however, when you write it up put it in your own words and make sure you understand it.) *You may use this result throughout the HW.*

3. (25 points)

- (a) Show there is an injection from $\{0, 1\}^\omega$ to $\{0, 1, 2\}^\omega$ (HINT: this is trivial).
- (b) Show there is an injection from $\{0, 1, 2\}^\omega$ to $\{0, 1\}^\omega$
- (c) From the two above statements what can you conclude?

4. (25 points) Let $PRIMES$ be the set of primes. Show that the set of all functions from \mathbb{N} to $PRIMES$ is uncountable.

5. (25 points) Let the set $Josh$ be defined as follows:

- If $p \in \mathbb{Z}[x]$ and α is any of the transcendental Numbers listed on the website of 15 awesome transcendental numbers (there is a pointer on the course website) then $p(\alpha)$ is in $Josh$.
- If p is a polynomial with integer coefficients and $n \in \mathbb{N}$, $n \geq 2$, then $p(\ln n)$ is in $Josh$.

Is $Josh$ countable or uncountable? Justify your answer.