

15-213

“The course that gives CMU its Zip!”

Cache Memories

Oct. 10, 2002

Topics

- Generic cache memory organization
- Direct mapped caches
- Set associative caches
- Impact of caches on performance

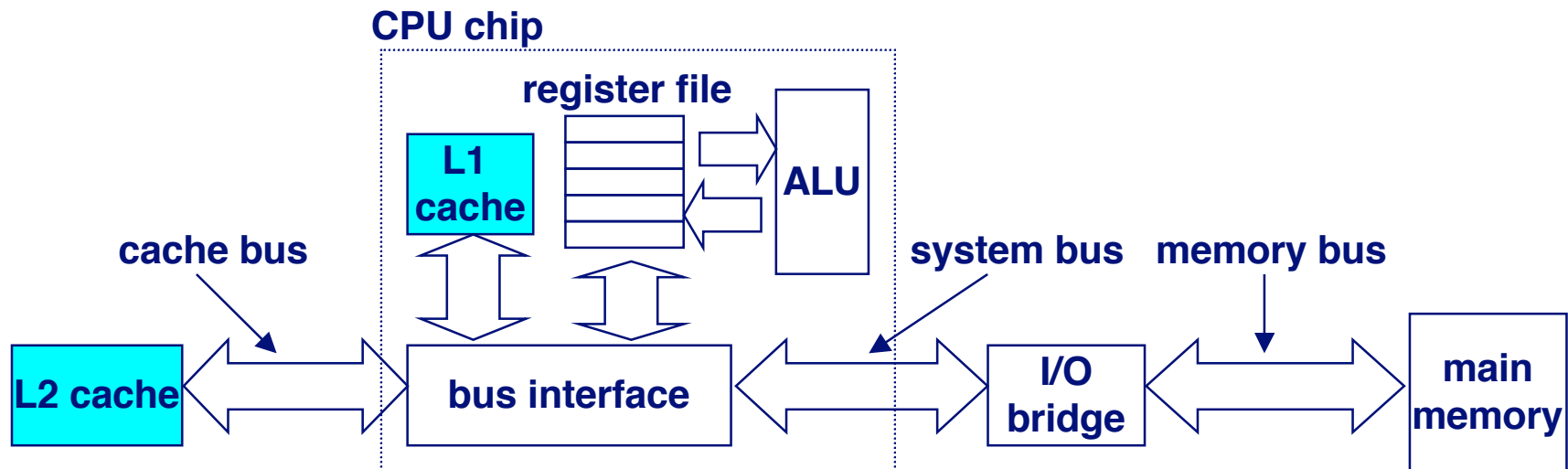
Cache Memories

Cache memories are small, fast SRAM-based memories managed automatically in hardware.

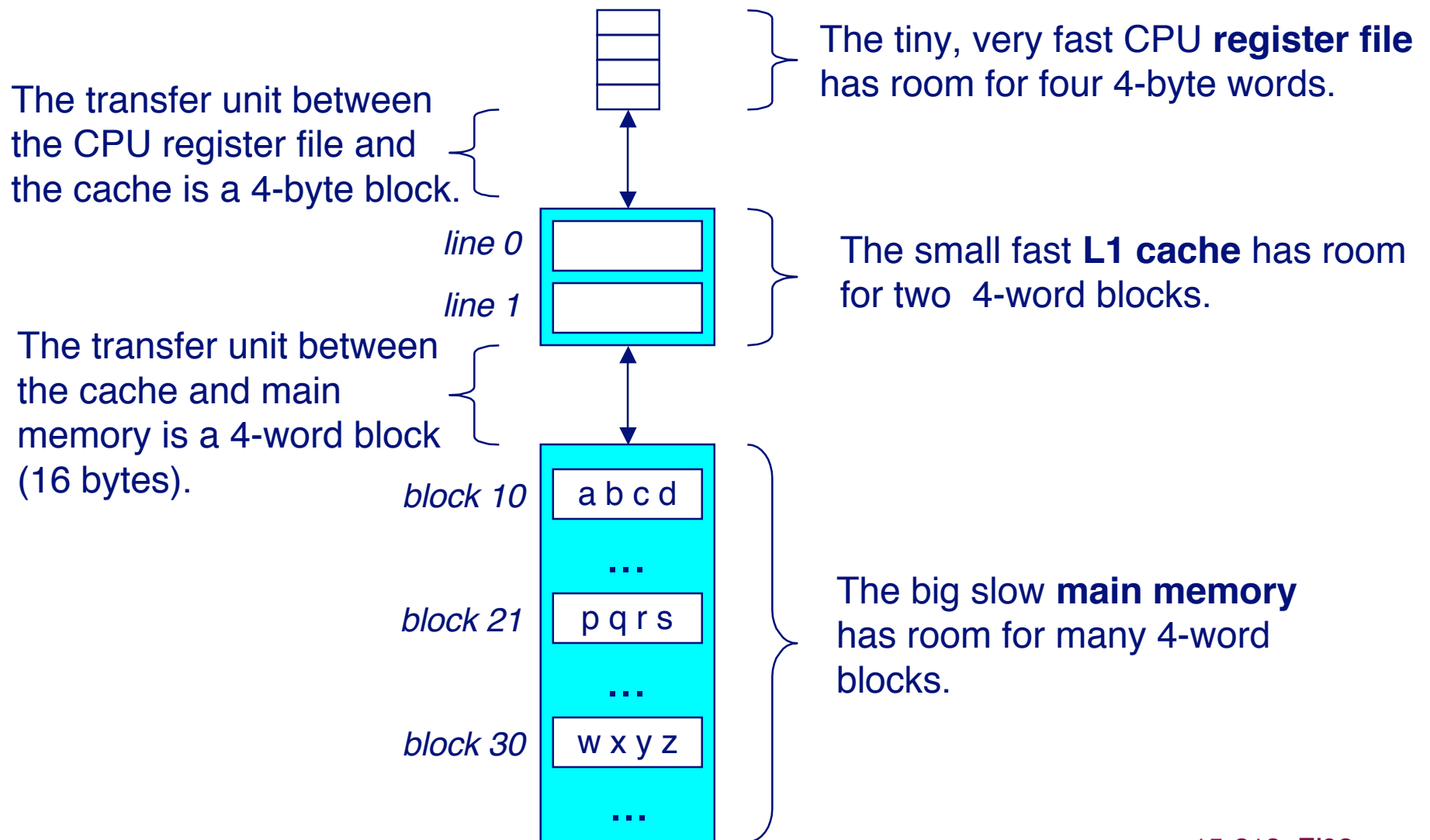
- Hold frequently accessed blocks of main memory

CPU looks first for data in L1, then in L2, then in main memory.

Typical bus structure:



Inserting an L1 Cache Between the CPU and Main Memory

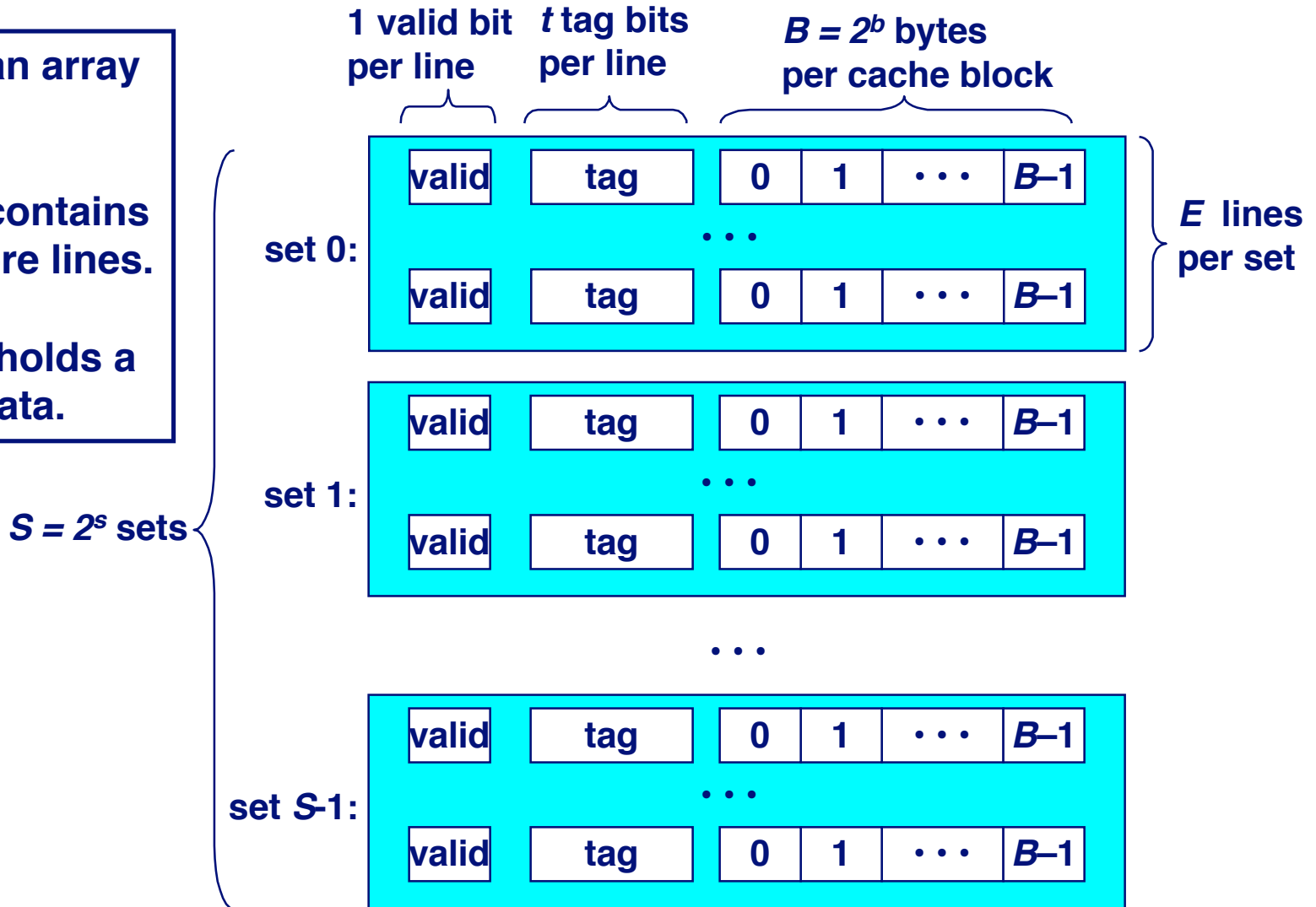


General Org of a Cache Memory

Cache is an array of sets.

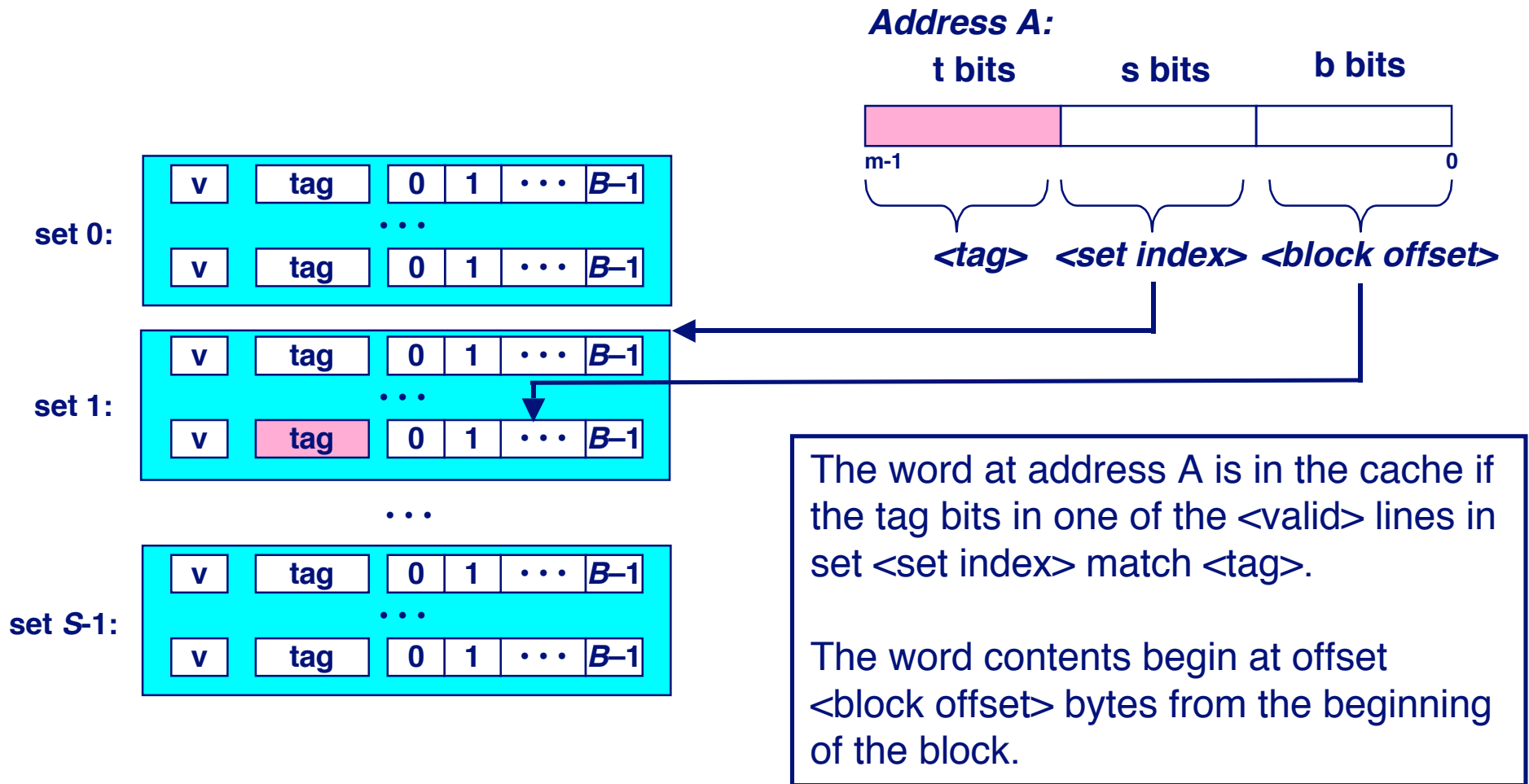
Each set contains one or more lines.

Each line holds a block of data.



Cache size: $C = B \times E \times S$ data bytes

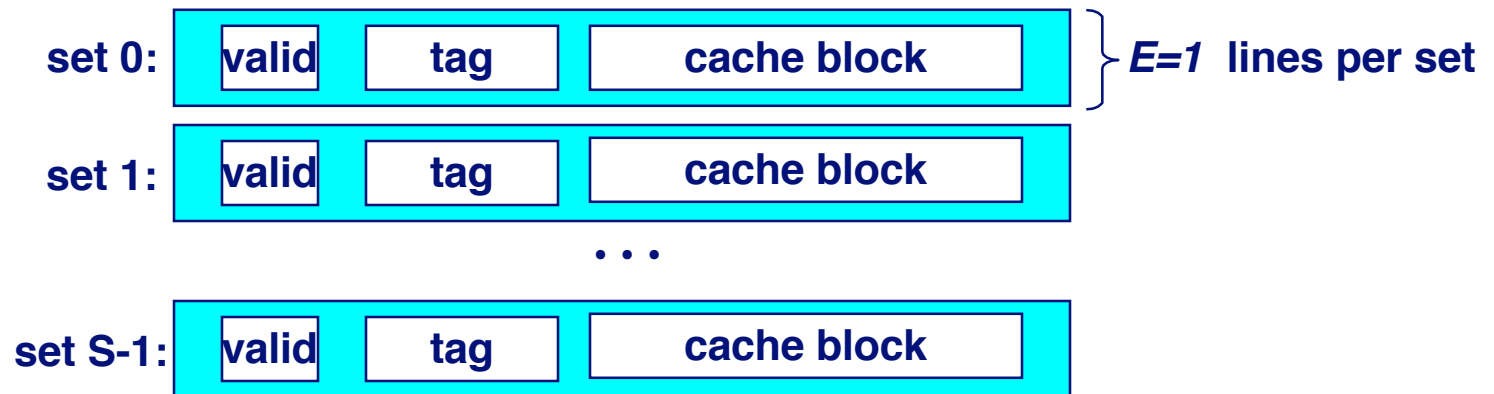
Addressing Caches



Direct-Mapped Cache

Simplest kind of cache

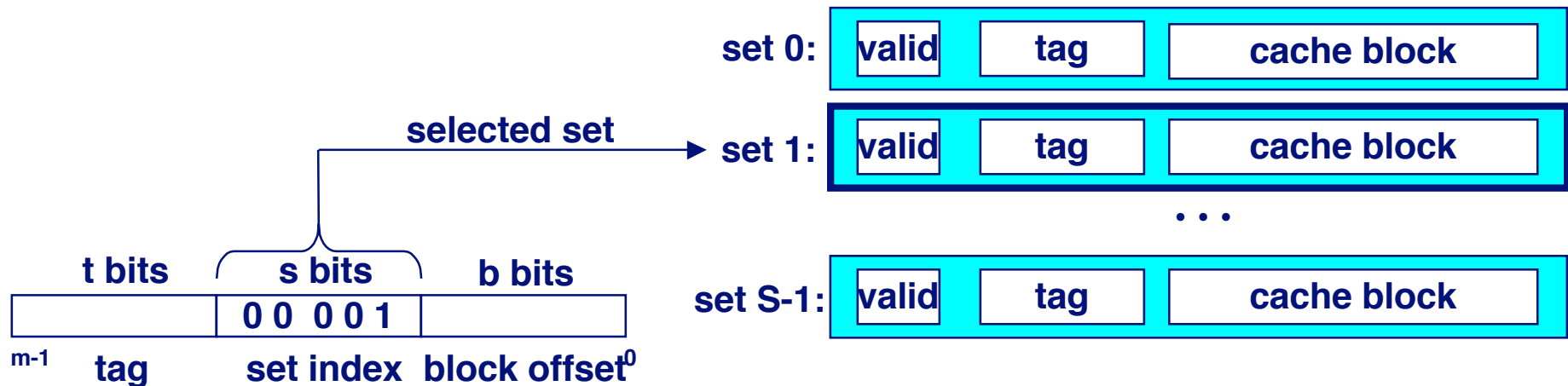
Characterized by exactly one line per set.



Accessing Direct-Mapped Caches

Set selection

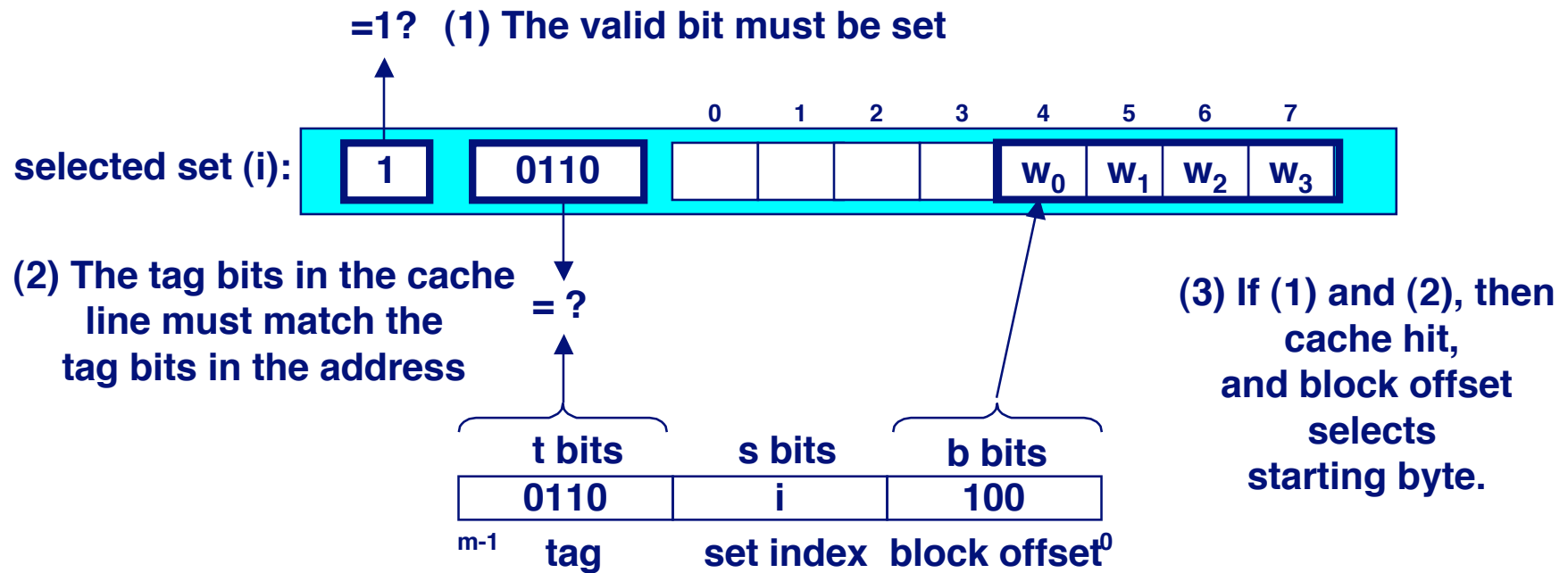
- Use the set index bits to determine the set of interest.



Accessing Direct-Mapped Caches

Line matching and word selection

- **Line matching:** Find a valid line in the selected set with a matching tag
- **Word selection:** Then extract the word



Direct-Mapped Cache Simulation

M=16 byte addresses, B=2 bytes/block,
S=4 sets, E=1 entry/set

t=1	s=2	b=1
X	XX	X

Address trace (reads):

0 [0000₂], 1 [0001₂], 13 [1101₂], 8 [1000₂], 0 [0000₂]

0 [0000₂] (*miss*)

v	tag	data
1	0	M[0-1]

(1)

13 [1101₂] (*miss*)

v	tag	data
1	0	M[0-1]
1	1	M[12-13]

(3)

8 [1000₂] (*miss*)

v	tag	data
1	1	M[8-9]
1	1	M[12-13]

(4)

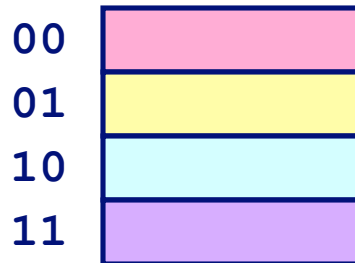
0 [0000₂] (*miss*)

v	tag	data
1	0	M[0-1]
1	1	M[12-13]

(5)

Why Use Middle Bits as Index?

4-line Cache



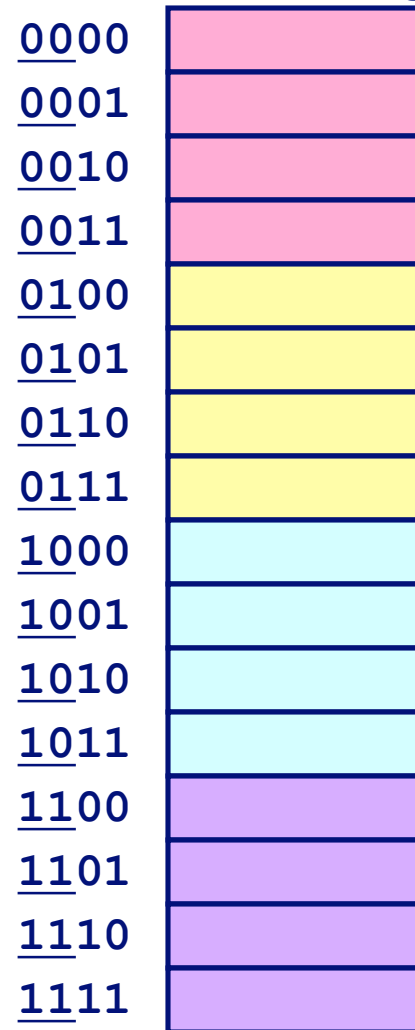
High-Order Bit Indexing

- Adjacent memory lines would map to same cache entry
- Poor use of spatial locality

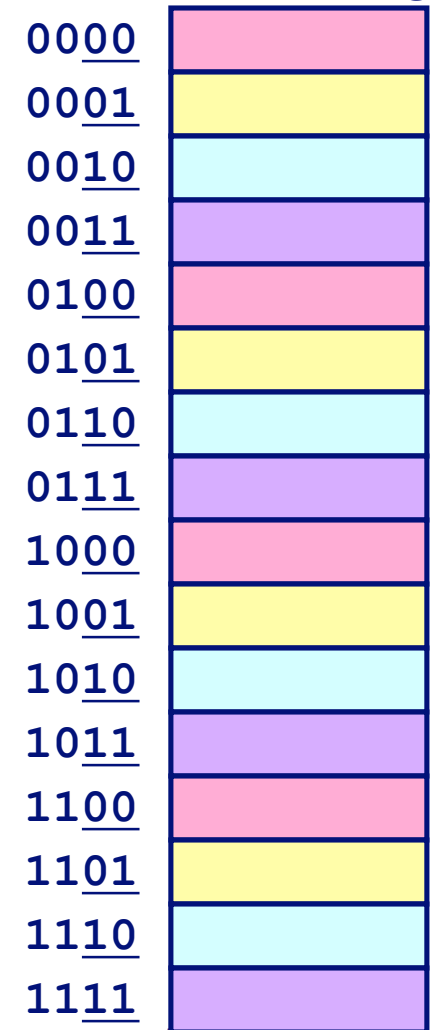
Middle-Order Bit Indexing

- Consecutive memory lines map to different cache lines
- Can hold C-byte region of address space in cache at one time

High-Order Bit Indexing



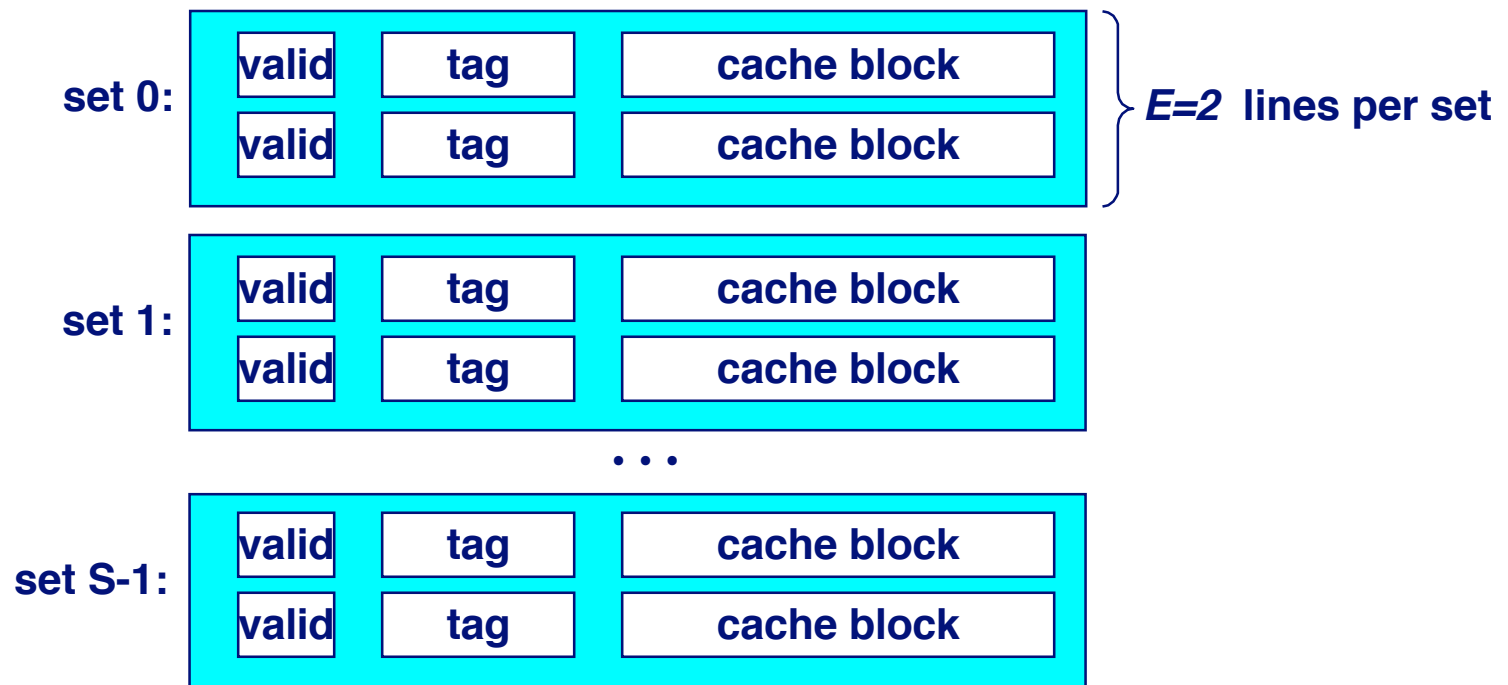
Middle-Order Bit Indexing



15-213, F02

Set Associative Caches

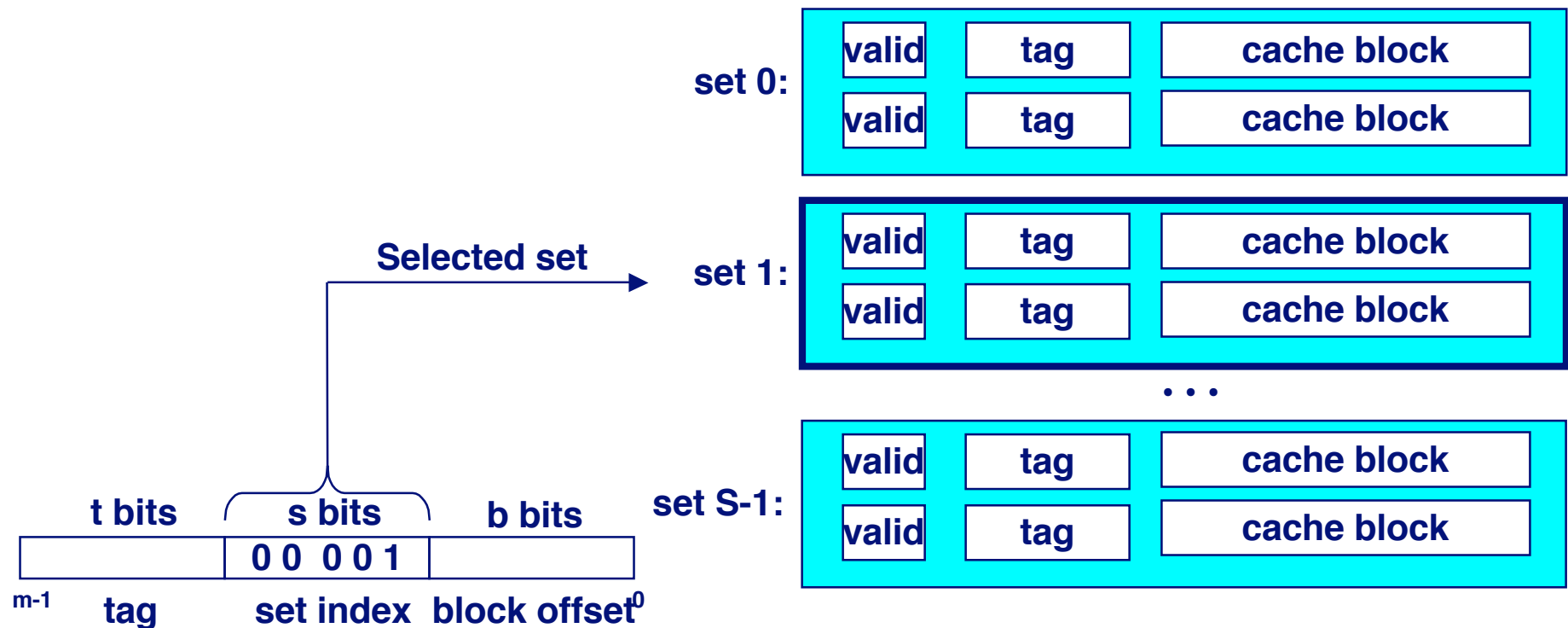
Characterized by more than one line per set



Accessing Set Associative Caches

Set selection

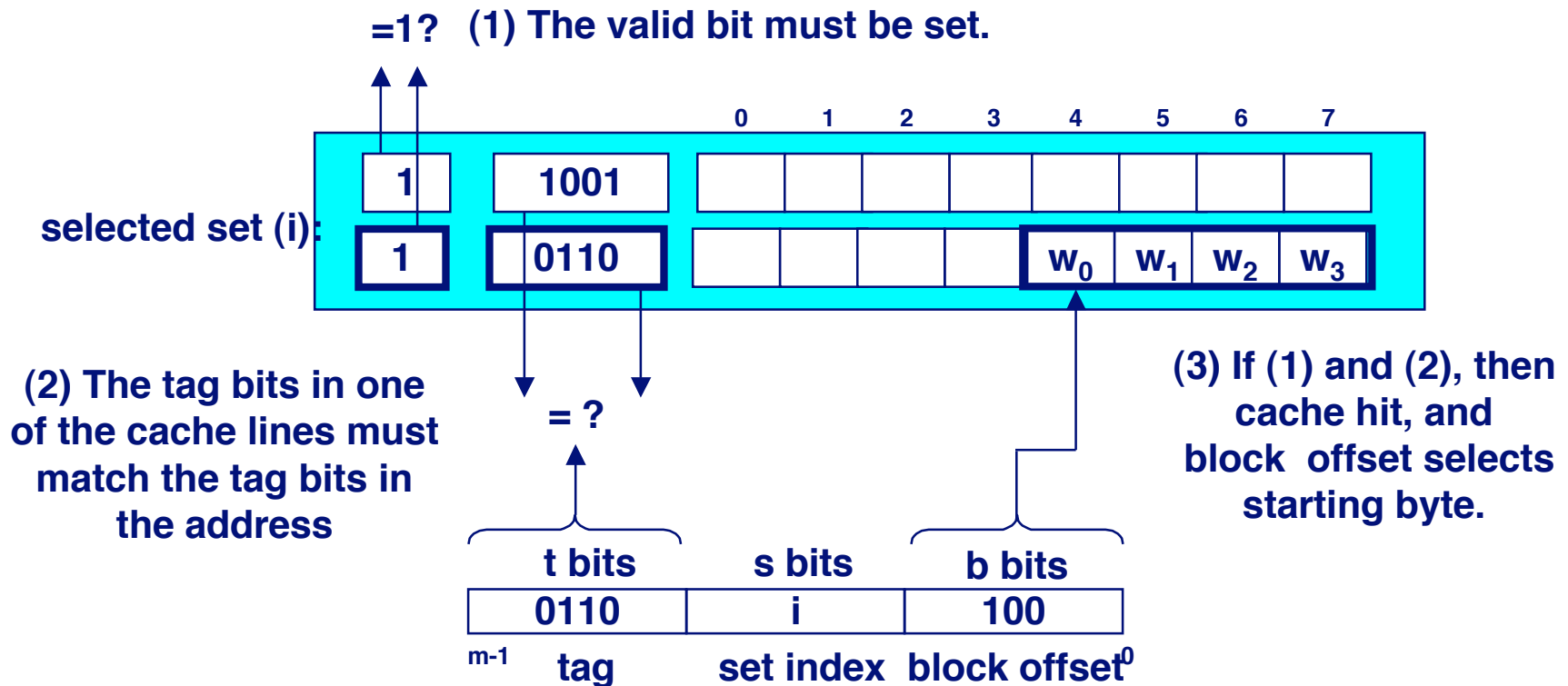
- identical to direct-mapped cache



Accessing Set Associative Caches

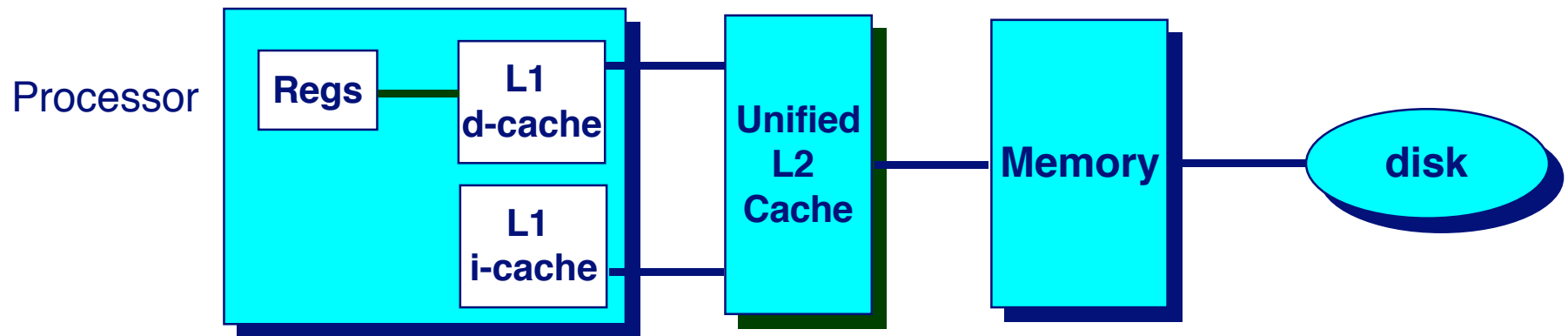
Line matching and word selection

- must compare the tag in each valid line in the selected set.



Multi-Level Caches

Options: separate **data** and **instruction caches**, or a **unified cache**

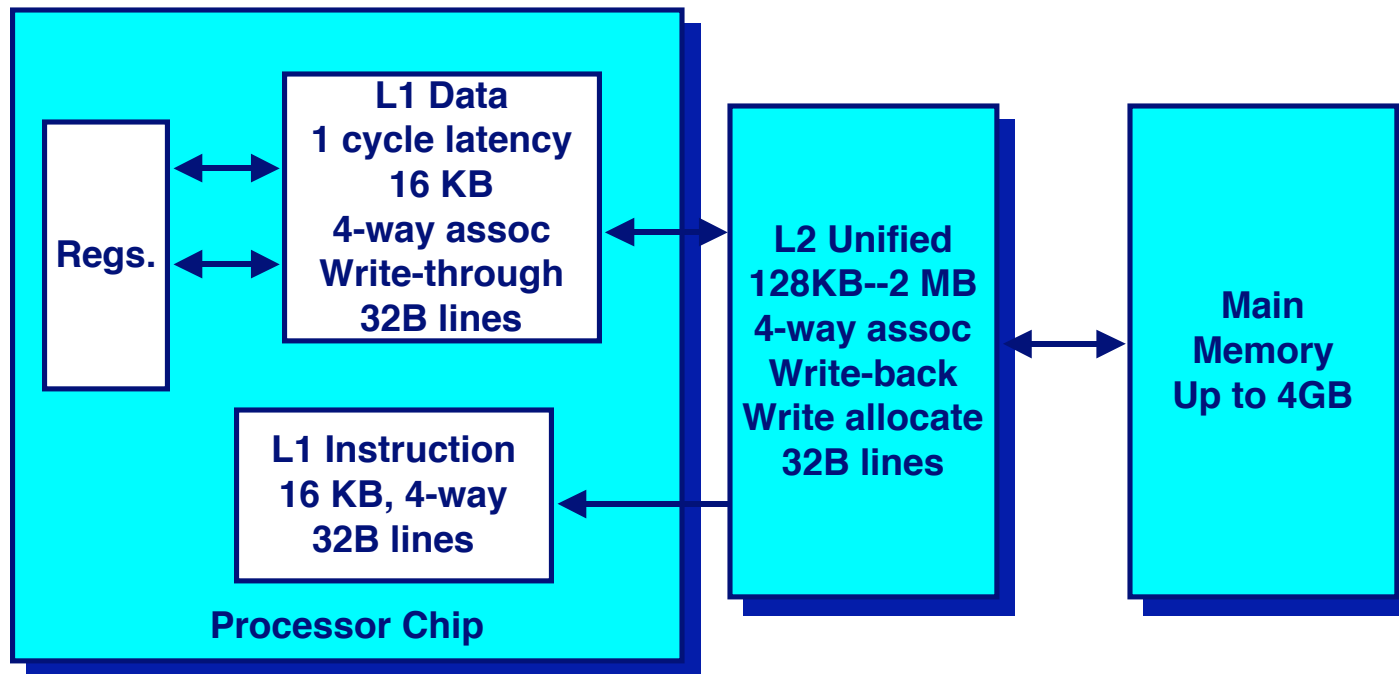


size:	200 B	8-64 KB	1-4MB SRAM	128 MB DRAM	30 GB
speed:	3 ns	3 ns	6 ns	60 ns	8 ms
\$/Mbyte:			\$100/MB	\$1.50/MB	\$0.05/MB
line size:	8 B	32 B	32 B	8 KB	

larger, slower, cheaper



Intel Pentium Cache Hierarchy



Cache Performance Metrics

Miss Rate

- Fraction of memory references not found in cache (misses/references)
- Typical numbers:
 - 3-10% for L1
 - can be quite small (e.g., < 1%) for L2, depending on size, etc.

Hit Time

- Time to deliver a line in the cache to the processor (includes time to determine whether the line is in the cache)
- Typical numbers:
 - 1 clock cycle for L1
 - 3-8 clock cycles for L2

Miss Penalty

- Additional time required because of a miss
 - Typically 25-100 cycles for main memory

Writing Cache Friendly Code

Repeated references to variables are good (temporal locality)

Stride-1 reference patterns are good (spatial locality)

Examples:

- cold cache, 4-byte words, 4-word cache blocks

```
int sumarrayrows(int a[M][N])
{
    int i, j, sum = 0;

    for (i = 0; i < M; i++)
        for (j = 0; j < N; j++)
            sum += a[i][j];
    return sum;
}
```

Miss rate = $1/4 = 25\%$

```
int sumarraycols(int a[M][N])
{
    int i, j, sum = 0;

    for (j = 0; j < N; j++)
        for (i = 0; i < M; i++)
            sum += a[i][j];
    return sum;
}
```

Miss rate = 100%

The Memory Mountain

Read throughput (read bandwidth)

- Number of bytes read from memory per second (MB/s)

Memory mountain

- Measured read throughput as a function of spatial and temporal locality.
- Compact way to characterize memory system performance.

Memory Mountain Test Function

```
/* The test function */
void test(int elems, int stride) {
    int i, result = 0;
    volatile int sink;

    for (i = 0; i < elems; i += stride)
        result += data[i];
    sink = result; /* So compiler doesn't optimize away the loop */
}

/* Run test(elems, stride) and return read throughput (MB/s) */
double run(int size, int stride, double Mhz)
{
    double cycles;
    int elems = size / sizeof(int);

    test(elems, stride); /* warm up the cache */
    cycles = fcyc2(test, elems, stride, 0); /* call test(elems, stride) */
    return (size / stride) / (cycles / Mhz); /* convert cycles to MB/s */
}
```

Memory Mountain Main Routine

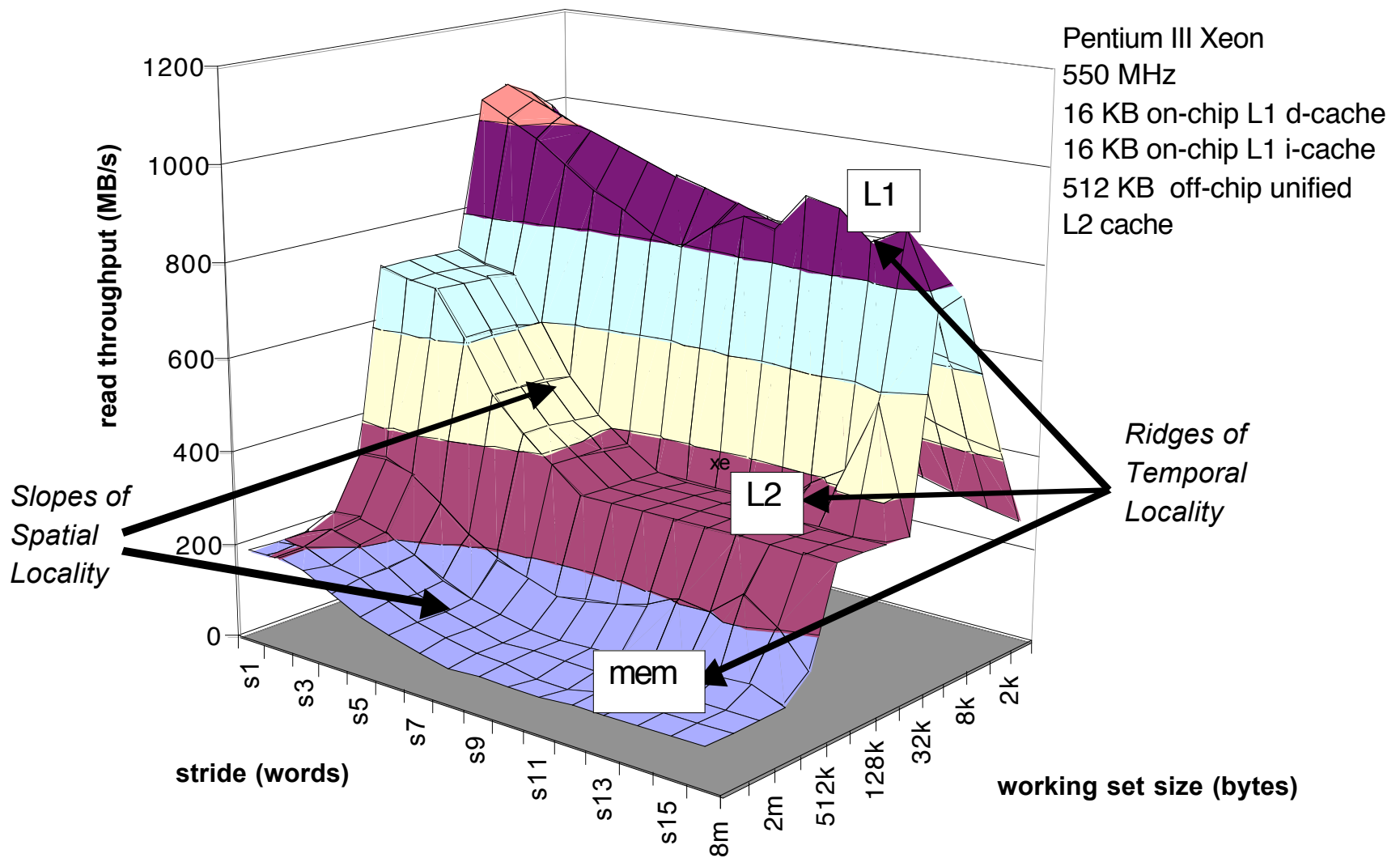
```
/* mountain.c - Generate the memory mountain. */
#define MINBYTES (1 << 10) /* Working set size ranges from 1 KB */
#define MAXBYTES (1 << 23) /* ... up to 8 MB */
#define MAXSTRIDE 16 /* Strides range from 1 to 16 */
#define MAXELEMS MAXBYTES/sizeof(int)

int data[MAXELEMS]; /* The array we'll be traversing */

int main()
{
    int size; /* Working set size (in bytes) */
    int stride; /* Stride (in array elements) */
    double Mhz; /* Clock frequency */

    init_data(data, MAXELEMS); /* Initialize each element in data to 1 */
    Mhz = mhz(0); /* Estimate the clock frequency */
    for (size = MAXBYTES; size >= MINBYTES; size >>= 1) {
        for (stride = 1; stride <= MAXSTRIDE; stride++)
            printf("%.1f\t", run(size, stride, Mhz));
        printf("\n");
    }
    exit(0);
}
```

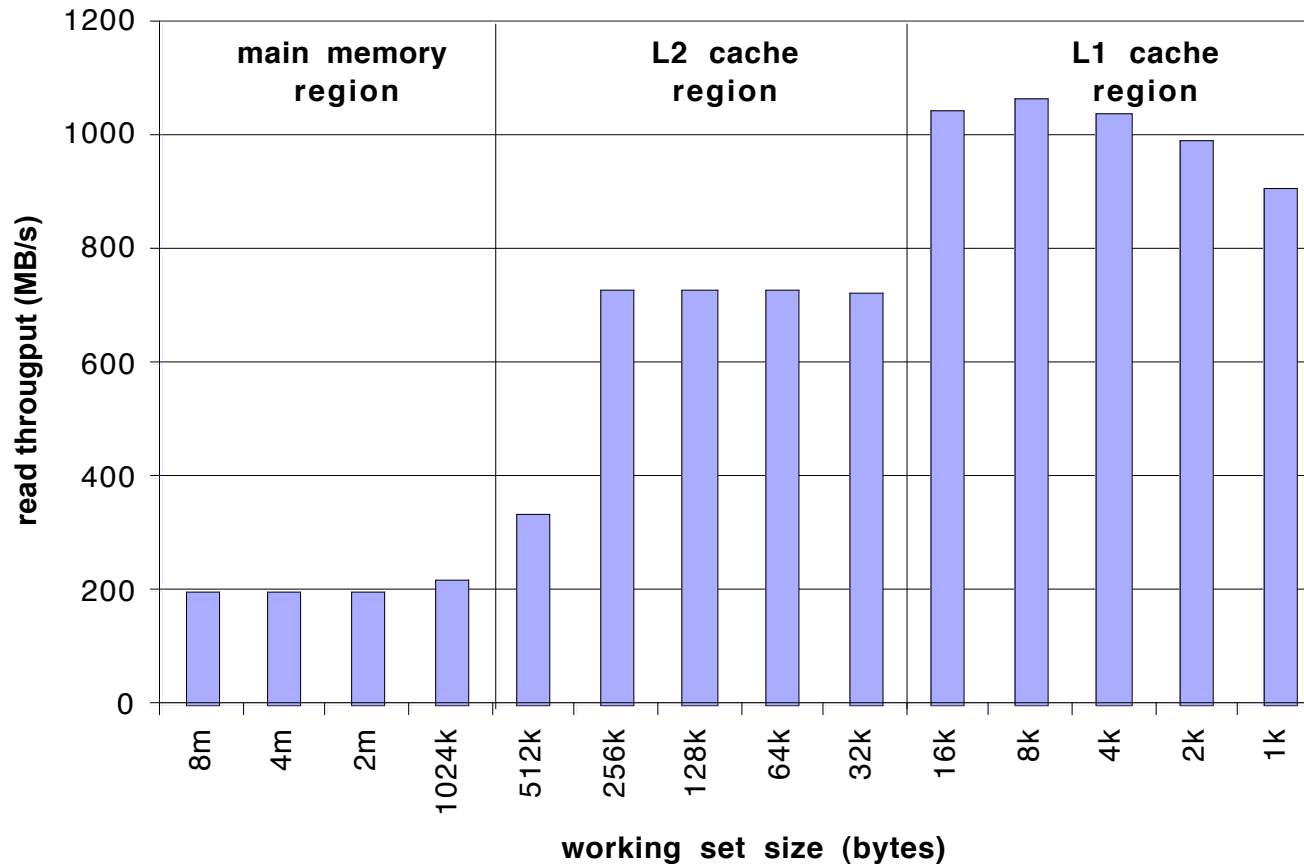
The Memory Mountain



Ridges of Temporal Locality

Slice through the memory mountain with stride=1

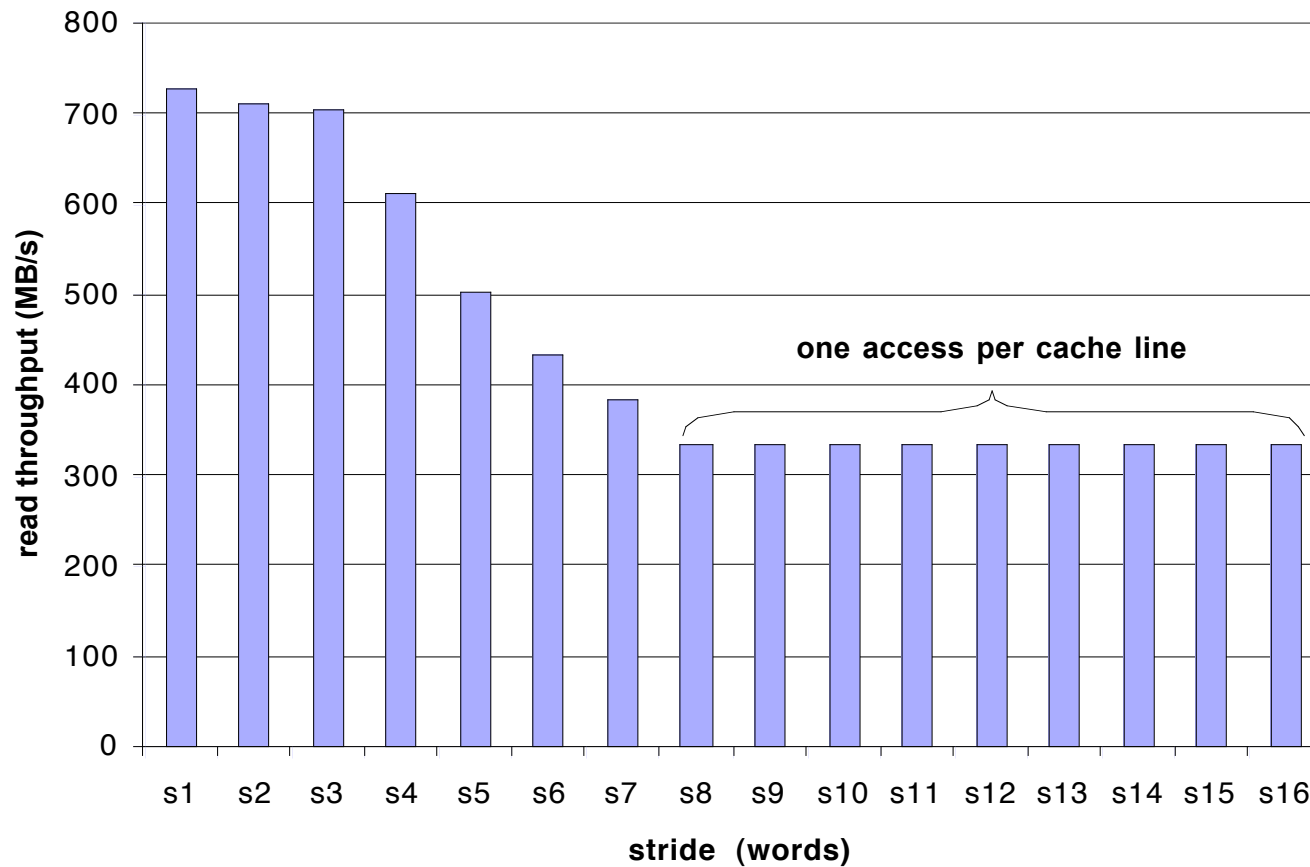
- illuminates read throughputs of different caches and memory



A Slope of Spatial Locality

Slice through memory mountain with size=256KB

■ shows cache block size.



Matrix Multiplication Example

Major Cache Effects to Consider

- Total cache size
 - Exploit temporal locality and keep the working set small (e.g., by using blocking)
- Block size
 - Exploit spatial locality

Description:

- Multiply $N \times N$ matrices
- $O(N^3)$ total operations
- Accesses
 - N reads per source element
 - N values summed per destination
 - » but may be able to hold in register

```
/* ijk */
for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

Variable sum held in register

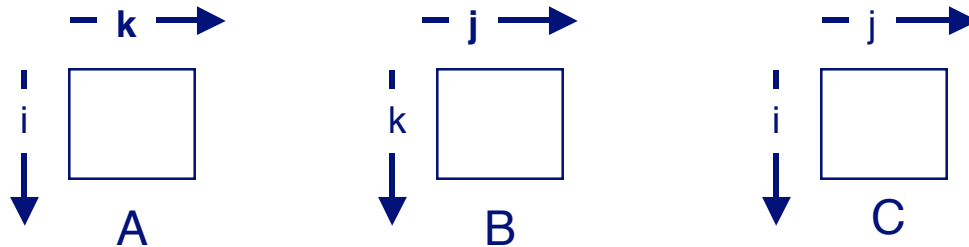
Miss Rate Analysis for Matrix Multiply

Assume:

- Line size = $32B$ (big enough for 4 64-bit words)
- Matrix dimension (N) is very large
 - Approximate $1/N$ as 0.0
- Cache is not even big enough to hold multiple rows

Analysis Method:

- Look at access pattern of inner loop



Layout of C Arrays in Memory (review)

C arrays allocated in row-major order

- each row in contiguous memory locations

Stepping through columns in one row:

- `for (i = 0; i < N; i++)`
 `sum += a[0][i];`
- accesses successive elements
- if block size (B) > 4 bytes, exploit spatial locality
 - compulsory miss rate = 4 bytes / B

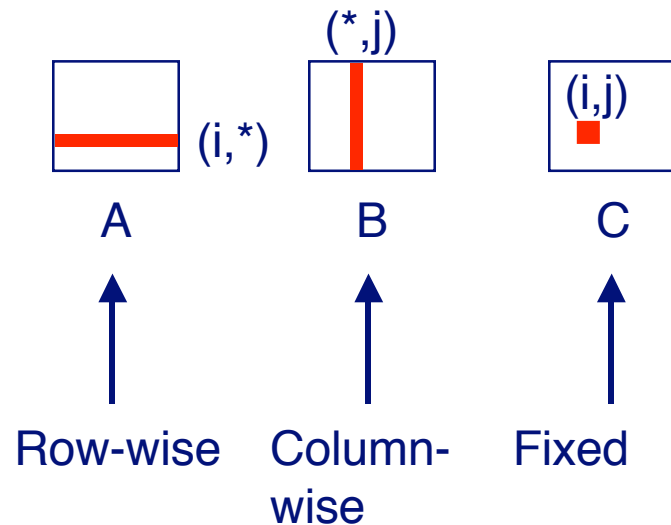
Stepping through rows in one column:

- `for (i = 0; i < n; i++)`
 `sum += a[i][0];`
- accesses distant elements
- no spatial locality!
 - compulsory miss rate = 1 (i.e. 100%)

Matrix Multiplication (ijk)

```
/* ijk */  
for (i=0; i<n; i++) {  
  for (j=0; j<n; j++) {  
    sum = 0.0;  
    for (k=0; k<n; k++)  
      sum += a[i][k] * b[k][j];  
    c[i][j] = sum;  
  }  
}
```

Inner loop:



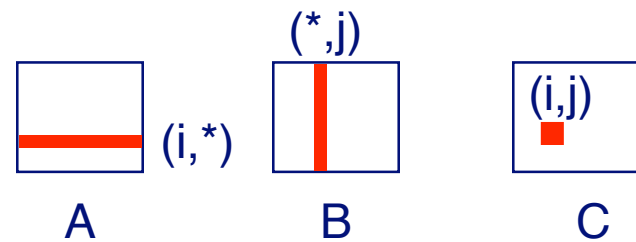
Misses per Inner Loop Iteration:

<u>A</u>	<u>B</u>	<u>C</u>
0.25	1.0	0.0

Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum
  }
}
```

Inner loop:



Row-wise

Column-wise

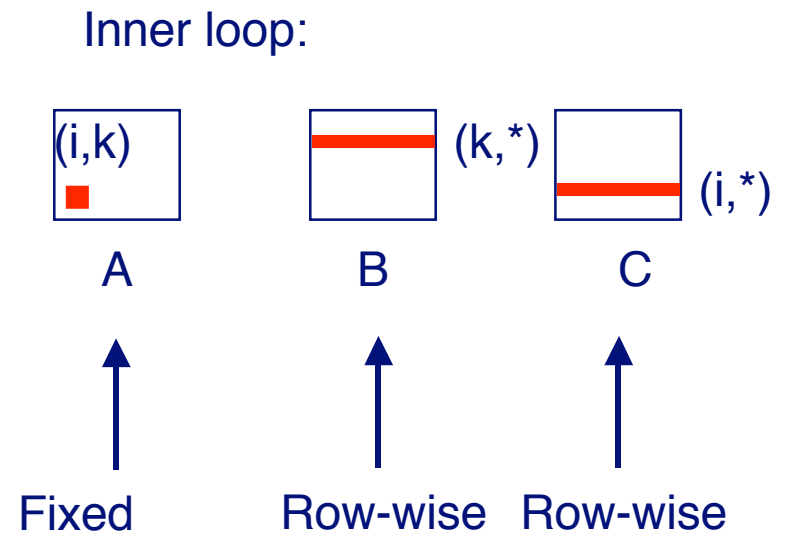
Fixed

Misses per Inner Loop Iteration:

<u>A</u>	<u>B</u>	<u>C</u>
0.25	1.0	0.0

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```



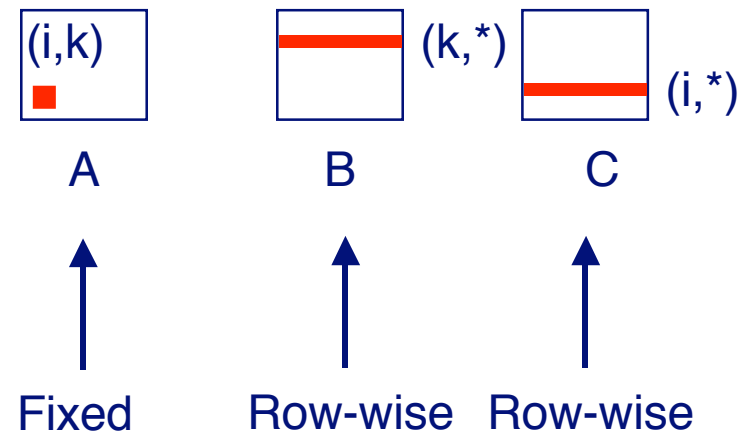
Misses per Inner Loop Iteration:

<u>A</u>	<u>B</u>	<u>C</u>
0.0	0.25	0.25

Matrix Multiplication (ikj)

```
/* ikj */  
for (i=0; i<n; i++) {  
  for (k=0; k<n; k++) {  
    r = a[i][k];  
    for (j=0; j<n; j++)  
      c[i][j] += r * b[k][j];  
  }  
}
```

Inner loop:



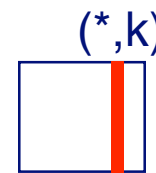
Misses per Inner Loop Iteration:

<u>A</u>	<u>B</u>	<u>C</u>
0.0	0.25	0.25

Matrix Multiplication (jki)

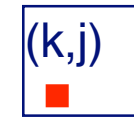
```
/* jki */  
for (j=0; j<n; j++) {  
  for (k=0; k<n; k++) {  
    r = b[k][j];  
    for (i=0; i<n; i++)  
      c[i][j] += a[i][k] * r;  
  }  
}
```

Inner loop:



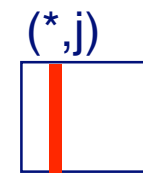
A

Column -
wise



B

Fixed



C

Column-
wise

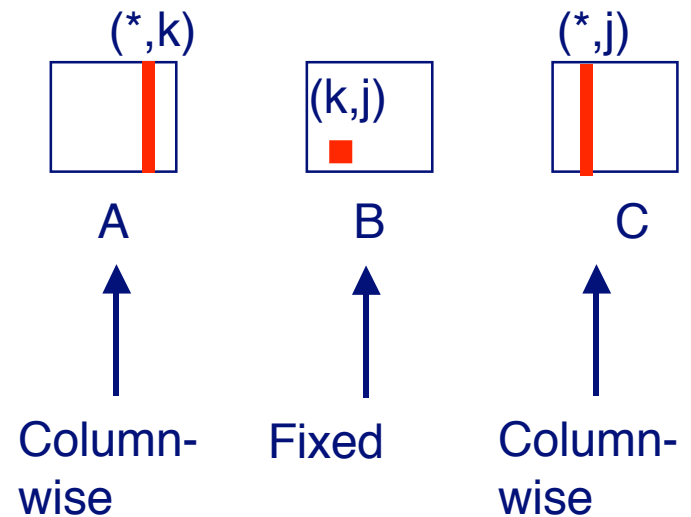
Misses per Inner Loop Iteration:

<u>A</u>	<u>B</u>	<u>C</u>
1.0	0.0	1.0

Matrix Multiplication (kji)

```
/* kji */  
for (k=0; k<n; k++) {  
  for (j=0; j<n; j++) {  
    r = b[k][j];  
    for (i=0; i<n; i++)  
      c[i][j] += a[i][k] * r;  
  }  
}
```

Inner loop:



Misses per Inner Loop Iteration:

<u>A</u>	<u>B</u>	<u>C</u>
1.0	0.0	1.0

Summary of Matrix Multiplication

ijk (& jik):

- 2 loads, 0 stores
- misses/iter = **1.25**

```
for (i=0; i<n; i++) {  
  for (j=0; j<n; j++) {  
    sum = 0.0;  
    for (k=0; k<n; k++)  
      sum += a[i][k] * b[k][j];  
    c[i][j] = sum;  
  }  
}
```

kij (& ikj):

- 2 loads, 1 store
- misses/iter = **0.5**

```
for (k=0; k<n; k++) {  
  for (i=0; i<n; i++) {  
    r = a[i][k];  
    for (j=0; j<n; j++)  
      c[i][j] += r * b[k][j];  
  }  
}
```

jki (& kji):

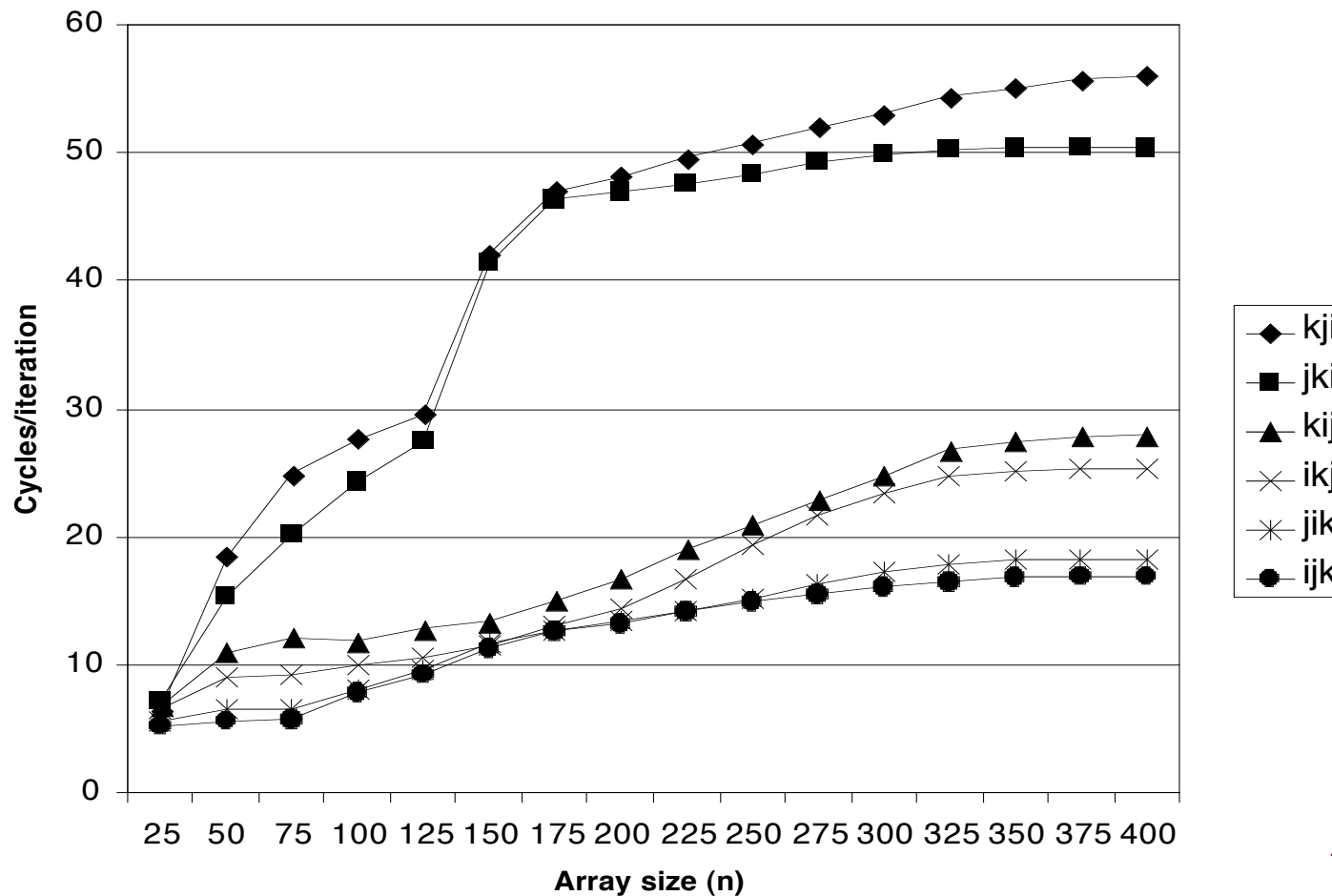
- 2 loads, 1 store
- misses/iter = **2.0**

```
for (j=0; j<n; j++) {  
  for (k=0; k<n; k++) {  
    r = b[k][j];  
    for (i=0; i<n; i++)  
      c[i][j] += a[i][k] * r;  
  }  
}
```

Pentium Matrix Multiply Performance

Miss rates are helpful but not perfect predictors.

- Code scheduling matters, too.



Improving Temporal Locality by Blocking

Example: Blocked matrix multiplication

- “block” (in this context) does not mean “cache block”.
- Instead, it means a sub-block within the matrix.
- Example: $N = 8$; sub-block size = 4

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

Key idea: Sub-blocks (i.e., A_{xy}) can be treated just like scalars.

$$C_{11} = A_{11}B_{11} + A_{12}B_{21} \quad C_{12} = A_{11}B_{12} + A_{12}B_{22}$$

$$C_{21} = A_{21}B_{11} + A_{22}B_{21} \quad C_{22} = A_{21}B_{12} + A_{22}B_{22}$$

Blocked Matrix Multiply (bijk)

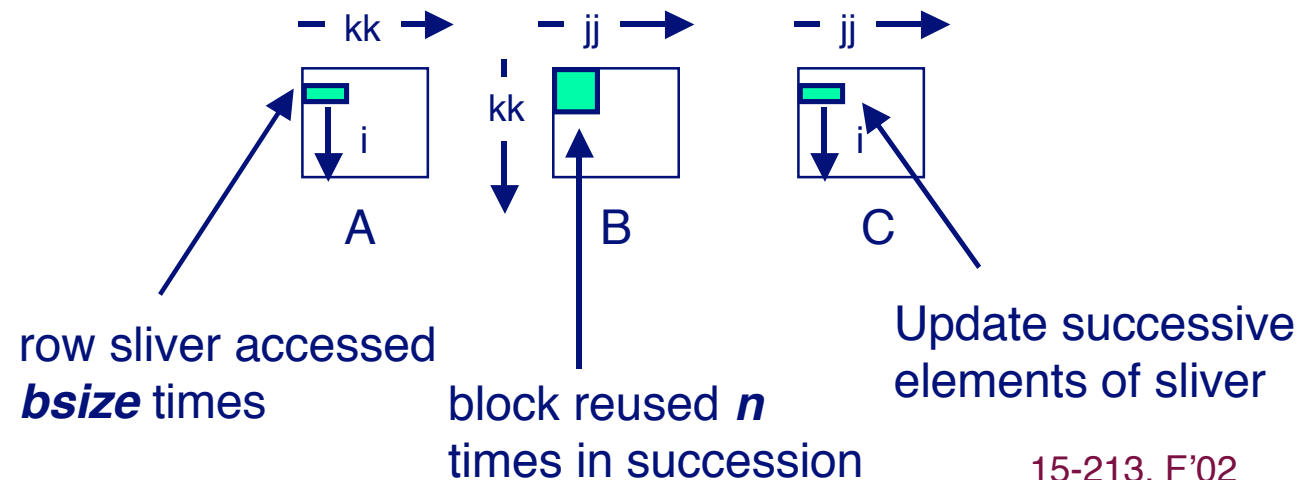
```
for (jj=0; jj<n; jj+=bsize) {
  for (i=0; i<n; i++)
    for (j=jj; j < min(jj+bsize,n); j++)
      c[i][j] = 0.0;
  for (kk=0; kk<n; kk+=bsize) {
    for (i=0; i<n; i++) {
      for (j=jj; j < min(jj+bsize,n); j++) {
        sum = 0.0
        for (k=kk; k < min(kk+bsize,n); k++) {
          sum += a[i][k] * b[k][j];
        }
        c[i][j] += sum;
      }
    }
  }
}
```

Blocked Matrix Multiply Analysis

- Innermost loop pair multiplies a $1 \times bsize$ sliver of A by a $bsize \times bsize$ block of B and accumulates into $1 \times bsize$ sliver of C
- Loop over i steps through n row slivers of A & C , using same B

```
for (i=0; i<n; i++) {  
    for (j=jj; j < min(jj+bsize,n); j++) {  
        sum = 0.0  
        for (k=kk; k < min(kk+bsize,n); k++) {  
            sum += a[i][k] * b[k][j];  
        }  
        c[i][j] += sum;  
    }  
}
```

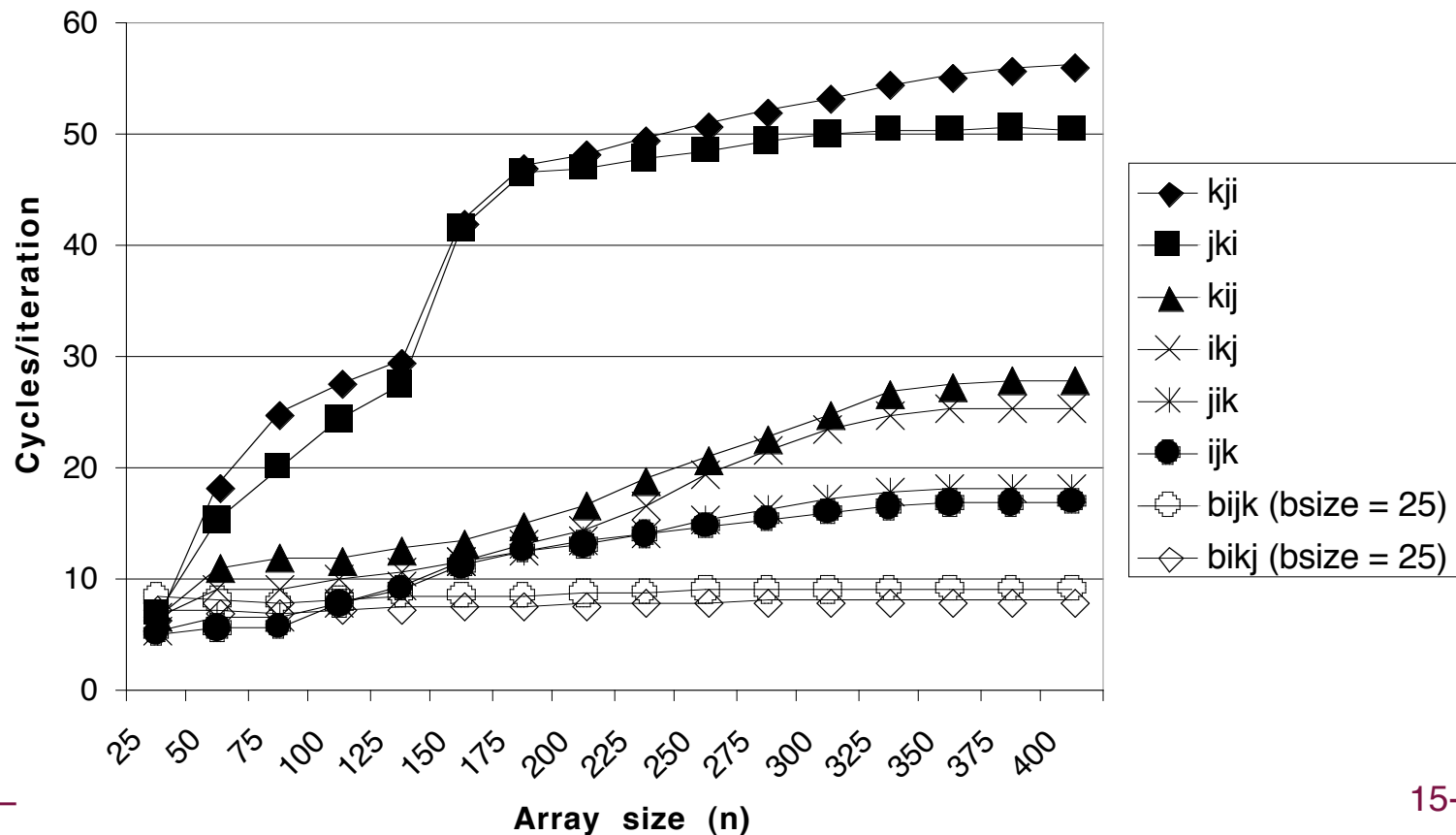
Innermost
Loop Pair



Pentium Blocked Matrix Multiply Performance

Blocking (bijk and bikj) improves performance by a factor of two over unblocked versions (ijk and jik)

- relatively insensitive to array size.



Concluding Observations

Programmer can optimize for cache performance

- How data structures are organized
- How data are accessed
 - Nested loop structure
 - Blocking is a general technique

All systems favor “cache friendly code”

- Getting absolute optimum performance is very platform specific
 - Cache sizes, line sizes, associativities, etc.
- Can get most of the advantage with generic code
 - Keep working set reasonably small (temporal locality)
 - Use small strides (spatial locality)