Joint Review [1] of
14 Book on Ramsey Theory

Review by
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1 Introduction

Some people collect Bibles. Some people collect First Editions. Some people collect First Editions of Bibles. I collect books on Ramsey Theory. I did not intend to do this. However, since I work in the field and I was the Sigact News book Review Editor, such books just happened to show up on my doorstep.

I told my graduate class in Ramsey Theory that I have 16 which is above the legal limit in Maryland. Some of them believed me.

The best way to straighten out in my own head what each book covers is to do a joint review of all 16 of them. *That Would Be Insane!* In this column I review those that mostly use techniques from combinatorics and are mostly elementary. I may, in later columns, review other sets of books.

2 Needed Background

Def 2.1 Let $k, n \in \mathsf{N}$.

1. $[n]$ is the set $\{1, \ldots, n\}$.

2. If A is a set then $\binom{A}{k}$ is the set of all k-sized subsetes of A.

3. Let COL$\colon \binom{A}{k} \to [c]$. A set $H \subseteq A$ is *homogenous (henceforth homog)* if COL restricted to $\binom{H}{k}$ is constant.

The following theorems are the first and most basic theorems in Ramsey Theory.

1. (Ramsey's Theorem [1]) For all a, c, k there exists $n = n(a, c, k)$ such that for all, COL$\colon \binom{[n]}{a} \to [c]$, there exists homog $H \subseteq [n]$ such that $|H| = k$.

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2. For all a, c, for all COL: $\binom{\mathsf{N}}{a} \to [c]$ there exists an infinite homog $H \subseteq \mathsf{N}$. such that $|H| = k$.

3. (van Der Waerden's Theorem [2]) For all c, k there $W = W(k, c)$ such that, for all COL: $[W] \to [k]$, there exists a monochromatic arithmetic sequence of length k.

3 Rudiments of Ramsey Theory, Second Edition, Graham and Butler. 1979, 2015

4 Ramsey Theory by Grahamm, Rothchild, Spencer. 1980, 1990

(First edition 1980, Second edition 1990)

5 Ramsey Theory on the Integers by Landman and Robertson. 2014

6 An Introduction to Ramsey Theory: Fast Functions, Infinity, And Metamathmatics by M. Katz and Reimann. 2018

7 Fundamentals of Ramsey Theory by Aaron Robertson. 2021

8 Basics of Ramsey Theory by Jungic. 2023

References

[1] F. Ramsey. On a problem of formal logic. *Proceedings of the London Math Society*, 30(1):264–286, 1930.

[2] B. van der Waerden. Beweis einer Baudetschen Vermutung (in dutch).
    Nieuw Arch. Wisk., 15:212–216, 1927.