1 ADD TO WHEN YOU FIRST DEFINE SHARP P

Empirically it seems that, for every natural problem A that is NP-complete, #A is #P-complete. This is not a theorem, and it probably cannot be a theorem since it is hard to define *natural*. Are there sets $A \in \mathrm{P}$ for which #A is #P-complete. Yes. We state two of them.

Theorem 1.1

1. *Brightwell et al. [1] showed that counting the number of Eulerian Circuits in a graph is* #P*-complete. Note that detecting if a graph has an Eucler Circuit is in* P*.*

2. *Jerrum [2] showed that counting the number of labeled trees in a graph is* #P*-complete. Note that detecting if some subgraph of a graph is a tree is trivial, the answer is always yes.*

3. *Valiant [6] showed that counting the number of bipartite matching is* #P*-complete. We will see this as an easy corollary of* PERM *being* #P*-complete. Note that finding a matching in a bipartite graph (even a general graph) is in* P*. Vadhan [5] showed that this problem is still* #P*-complete when G is restricted to bipartite graphs (1) with degree 4, (2) planar of degree 6, or (3) k-regular for any $k \geq 5$. That paper has many other problems in* P *whose counting version if* #P*-complete.*

4. *Valiant [6] showed that counting the number of Satisfying assignments in a montone 2-SAT formula is* #P*-complete. Note that the problem of determining if a monotone 2-SAT formula is satisfied is trivial, the answer is always yes. Provan and Ball [3] extended this result.*

2 Further Results

We list several other problems that are #P-complete.

- Counting the number of vertex covers of size $\leq k$ in a bipartite Vertex Cover. This was shown by Provan and Ball [3]. They also showed several variants of this problem are #P-complete.

- Minimum Cardinality (s,t) Cut: Given (G,s,t) where G is a graph and s,t are vertices, find how many minimum-size edge cuts separating s and t. This was shown by Pravan and Ball [3].

- Antichain. Given partial order $(X,\preceq)$ find the number of antichains (sets with all elements pairwise incomparable). This was shown by Provan and Ball [3]. They also showed several variants of the problem are #P-complete.

3 Mention in the ASP section

Fillmat - Fillmat is a logic puzzle published by Nikoli. It was shown to be both NP- and ASP-complete by Uejima and Suzuki [4].

References

[1] G. R. Brightwell and P. Winkler. Counting eulerian circuits is #p-complete. In C. Demetrescu, R. Sedgewick, and R. Tamassia, editors, *Proceedings of the Seventh Workshop on Algorithm Engineering and Experiments and the Second Workshop on Analytic Algorithmics and Combinatorics, ALENEX /ANALCO 2005, Vancouver, BC, Canada, 22 January 2005*, pages 259–262. SIAM, 2005.
`http://www.siam.org/meetings/analco05/papers/09grbrightwell.pdf`.

[2] M. Jerrum. Counting trees in a graph is #p-complete. *Inf. Process. Lett.*, 51(3):111–116, 1994.
`https://doi.org/10.1016/0020-0190(94)00085-9`.

[3] J. S. Provan and M. O. Ball. The complexity of counting cuts and of computing the probability that a graph is connected. *SIAM J. Comput.*, 12(4):777–788, 1983.
`https://doi.org/10.1137/0212053`.

[4] A. Uejima and H. Suzuki. Fillmat is np-complete and asp-complete. *J. Inf. Process.*, 23(3):310–316, 2015.
`https://doi.org/10.2197/ipsjjip.23.310`.

[5] S. P. Vadhan. The complexity of counting in sparse, regular, and planar graphs. *SIAM J. Comput.*, 31(2):398–427, 2001. `https://doi.org/10.1137/S0097539797321602`.

[6] L. G. Valiant. The complexity of enumeration and reliability problems. *SIAM Journal of Computing*, 8(3):410–421, 1979. `http://dx.doi.org/10.1137/0208032`.