BILL, RECORD LECTURE!!!!

BILL RECORD LECTURE!!!

Nondeterministic Finite Automata (NFA)

An Interesting Example of a DFA

With neighbor find DFA's for the following. Note numb. states.
$\Sigma^* a$
$\Sigma^* a \Sigma$
$\Sigma^* a \Sigma^2$

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https://www.cs.umd.edu/users/gasarch/COURSES/452/S21/notes/dfa3.JPG

$\Sigma^* a \Sigma^2$

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The number of states is 8.

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$\Sigma^* a \Sigma^i$ can be done with 2^{i+1} states.

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Prove for $\Sigma^* a \Sigma^3$, with a table.

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Might be on $2^{\{\text{HW, MIDTERM, FINAL}\}}$.

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We may prove this later.

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We now use NFA's informally.

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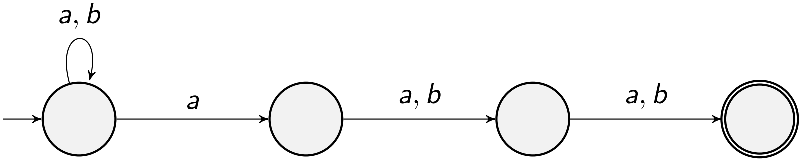
1. From a state q and a symbol σ there may be ≥ 2 states to go to.
2. From a state q and no symbols there may be ≥ 1 states to go to. (We use e for **empty string**.)
3. An NFA accepts a string if there is **some** way to process the string and get to a final state.

NFA for $\Sigma^* a \Sigma^2$



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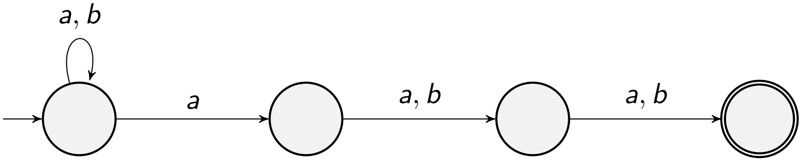

DFA had 8 states. NFA has 4 states.

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Draw an NFA for $\Sigma^* a \Sigma^3$.

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Make a conjecture for number of states for NFA for $\Sigma^* a \Sigma^n$.

Upshot Seems like NFA uses far fewer state than DFA for $\Sigma^* a \Sigma^n$.

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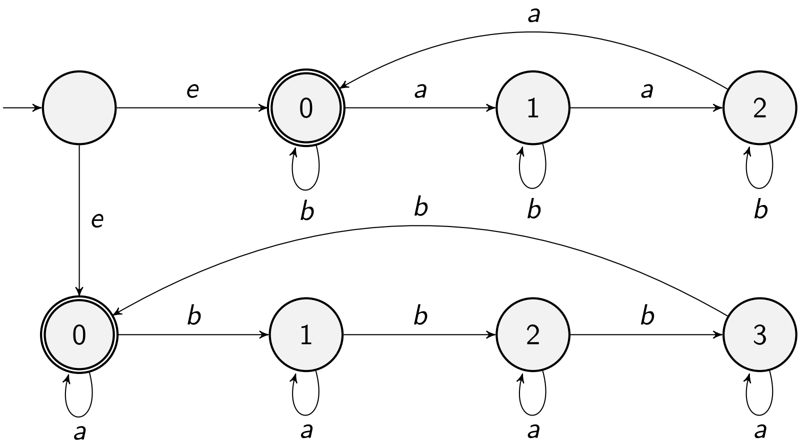

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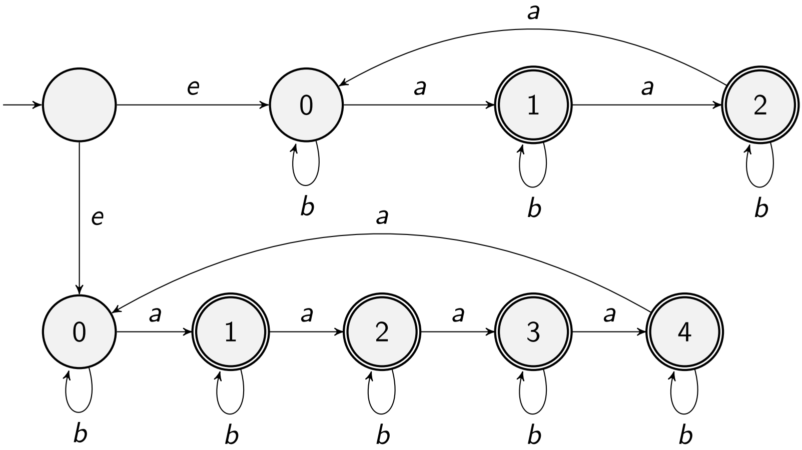

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Prove that the NFA in the last slide works.
Need

$$(n \not\equiv 0 \pmod 3 \vee n \not\equiv 0 \pmod 5) \implies n \not\equiv 0 \pmod{15}$$

Take the contrapositive

$$n \equiv 0 \pmod{15} \implies (n \equiv 0 \pmod 3 \wedge n \equiv 0 \pmod 5)$$

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NFA's Intuitively

1. An NFA is a DFA that can guess.
2. NFAs do not really exist.
3. Good for $\cup$ since can guess which one.
4. An NFA accepts iff SOME guess accepts.

NFA Formally

Def An **NFA** is a tuple $(Q, \Sigma, \Delta, s, F)$ where:

1. Q is a finite set of **states**.
2. Σ is a finite **alphabet**.
3. $\Delta : Q \times (\Sigma \cup \{e\}) \rightarrow 2^Q$ is the *transition function*.
4. $s \in S$ is the **start state**.
5. $F \subseteq Q$ is the set of **final states**.

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Note When you run $M(x)$ and choose a path one of three things can happen: (1) ends in a final state, (2) ends in a non-final state, (3) cannot process.

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Def If M is an NFA then $L(M) = \{x : M(x) \text{ accepts } \}$.

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- **Computational (with parallelism):** Fork new computational threads whenever there is a choice. Accept if any thread accepts.
- **Mathematical:** Create tree with branches whenever there is a choice. Accept if any leaf accepts.
- **Magic:** Guess at each nondeterministic step which way to go. Machine always makes right guess if there is one.

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We are nowhere near done. Next slide.

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Thm If L is accepted by an NFA with n states and no e-transitions then L is accepted by a DFA with $\leq 2^n$ states.
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