Misc Context Free Languages Stuff

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1. If L is a Context Free Langauge then there is a CFL for it in Chomsky Normal Form.
2. If $w \in \Sigma^*$ and $|w| = n$ then there is a CFL for w with $O(n)$ rules.
3. Pumping Theorem for CFL.

Every CFL has a CFG in CNF

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1) $A \rightarrow BC$ where $A, B, C \in N$ (nonterminals).
2) $A \rightarrow \sigma$ (where $A \in N$ and $\sigma \in \Sigma$).
3) $S \rightarrow e$ (where S is the start state).

Main Theorem about Chomsky Normal Form

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$S \rightarrow aabaA$

Transforming . . .

$S \rightarrow aabaA$ **Done**

Transforming ...

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$A \rightarrow SBaa$ **Can Use Same Method as on** $S \rightarrow aabaA$

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Transforming $A \rightarrow B$

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Discuss How to deal with $A \to B$?

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$S \to aabaA$ **Add** $S \to aabaB$

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$B \rightarrow bAAb$ **Add** $B \rightarrow bBAb \mid bABb \mid bBBb$

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2. For every rule of the form $X \to Y$ do what I did for $A \to B$.

CNF for $\{w\}$

CNF for $\{aabbbab\}$

Example CNF for $\{aabbbab\}$

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$S \rightarrow [A][ABBBAB]$

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We will return to this question later in the course.

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3. For all $i \geq 0$, $uv^i xy^i z \in L$.

Proof involves looking at the Parse Tree for w and finding some nonterminal T twice in the tree. We will not be doing the proof.

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Theorem Let $L \subseteq a^*$. If L is not regular then L is not a CFL.