HW08 Solution

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$$(z_{i,j,\sigma} \wedge z_{i,j+1,(q,a)}) \to (z_{i+1,j,(p,\sigma)} \wedge z_{i+1,j+1,(p,b)})$$

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$T(n)$ should be of the form $2^{O(n^c)}$ for a c that depends on a, b.

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$T(n) = 2^{n^a} \times 2^{O(n^{ab})} = 2^{O(n^{ab})}$.