Homework 8 Morally Due April 16 at 3:30PM

1. (30 points) Show that NP is closed under intersection.

2. (40 points) In this problem we will look at instructions a Turing Machine could have and how they would be modeled by Boolean Formulas (by modifying the proof of the Cook-Levin Theorem)

   We will DO one such problem and then assign two others.

   (a) (0 points- I give the answer so that you can do the other parts more easily) Assume that a Turing Machine has an instruction of the following type:

   $$\delta(q, a) = (p, RR)$$

   which means that if the head is looking at the symbol a, and the state is q, then the head of the Turing Machine moves TWO steps to the right and the state changes to p. Nothing on the tape changes, though the configuration will change since the head has moved.

   Give the formula that models this instruction.

   ANSWER:

   We first look at what happens if the configuration is $(q, a)bb$? Here is the sequence of parts of the configurations.

(q, a)	b	b
a	b	(p, b)

   The formula is

   $$(z_{i,j,(q,a)} \wedge z_{i,j+1,b} \wedge z_{i,j+2,b}) \rightarrow (z_{i+1,j,a} \wedge z_{i+1,j+1,b} \wedge z_{i+1,j+2,(p,b)})$$

   This is NOT the final answer since the configuration could have other symbols where I have the bb. Here is what happens if the config is $(q, a)\sigma_1\sigma_2$.

(q, a)	σ_1	σ_2
a	σ_1	(p, σ_2)

   Hence the formula is

$$\bigwedge_{(\sigma_1,\sigma_2)\in\Sigma\times\Sigma}$$

$$\left(z_{i,j,(q,a)} \wedge z_{i,j+1,\sigma_1} \wedge z_{i,j+2,\sigma_2}\right) \rightarrow \left(z_{i+1,j,a} \wedge z_{i+1,j+1,\sigma_1} \wedge z_{i+1,j+2,(p,\sigma_2)}\right)$$

When you answer the questions below you NEED to have both a diagram like this one:

(q,a)	σ_1	σ_2
a	σ_1	(p,σ_2)

and the formula like this one:

$$\bigwedge_{(\sigma_1,\sigma_2)\in\Sigma\times\Sigma}$$

$$\left(z_{i,j,(q,a)} \wedge z_{i,j+1,\sigma_1} \wedge (z_{i,j+2,\sigma_2}\right) \rightarrow \left(z_{i+1,j,a} \wedge z_{i+1,j+1,\sigma_1} \wedge z_{i+1,j+2,(p,\sigma_2)}\right)$$

And NOW for the problems YOU need to do.

(b) (20 points) Do what I did above for the transition

$\delta(q,a) = (p,b,L)$

which means that if the head is looking at an a and the machine is in state q then it will overwrite the a it is looking with by a b **AND** change state to p **AND** move Left.

(c) (20 points) Do what I did above for the transition

$\delta(q,a) = (p,L,b)$

which means that if the head is looking at an a and the machine is in state q then it will move Left **AND THEN** write a b in the square (overwriting whatever was there) **AND THEN** change state to p.

3. (30 points) Let $a, b \in \mathsf{N}$, $a, b \geq 2$. Let B be solvable in time 2^{n^b} time where n is the length of the input to B. (NOTE- in this problem the input to B will be an ordered pair (x, y) where $|x| = n$ and $|y| = n^a$ (as you will see soon). since $n \ll n^a$ we will consider the length of the input to B to be $O(n^a)$.) Let

$$A = \{x : (\exists y, |y| = |x|^a)[(x, y) \in B].$$

Give an algorithm that determines if $x \in A$. Give $T(n)$, the time bound on the algorithm for inputs of length n. $T(n)$ should be of the form $2^{O(n^c)}$ for a c that depends on a, b.