Homework 4 Morally Due Feb 27 at 3:30PM

1. (0 points, but if you actually miss the midterm without telling Dr. Gasarch ahead of time, you will lose 100 points on this homework) When will the midterm be (give date and time)?

2. (40 points-10 points each) For each of the following languages:

- State REGULAR or NOT REGULAR. This should be the FIRST THING on your paper. (In the past some guy wrote incoherent things so I could not tell which direction they are trying to prove. DON'T BE THAT GUY!)
- If REGULAR than give a DFA or NFA or regex or textbook regex for it. The DFA or NFA may be given as either a picture or a transition table. You MAY use DOT DOT DOT.
- If NOT REGULAR than PROVE IT. You MAY use DOT DOT DOT.

(a) $L_1 = \{a^n a^n : n \geq 1000\}$.

(b) $L_2 = \{a^n a^n : n \leq 1000\}$.

(c) $L_3 = \{a^{\lfloor \log_2 n \rfloor} : n \geq 1\}$.

(d) For this problem $\Sigma = \{a, b\}$. RECALL that $\#_\sigma(w)$ is the number of σ's in w. We generalize this to allow σ to be a string.

Definition $\#_{ab}(w)$ is the number of times the string ab occurs in w. $\#_{ba}(w)$ is the number of times the string ba occurs in w. (We can replace ab and ba with any element of Σ^*.)

$$L_4 = \{w : \#_{ab}(w) = \#_{ba}(w)\}.$$

Examples
$aba \in L_4$ since $\#_{ab}(w) = 1$ and $\#_{ba}(w) = 1$.
$abaaaaaab \notin L_4$ since $\#_{ab}(w) = 2$ and $\#_{ba}(w) = 1$.

3. (30 points–10 each) For each of the following languages show that they are NOT regular.

(a) $L_5 = \{w : \#_a(w) = 2\#_b(w)\}$.

(b) $L_6 = \{w : 2\#_a(w) = 3\#_b(w)\}$.

(c) Let $c, d \in \mathsf{N}$, $c, d \geq 1$.

$$L_7 = \{w : c\#_a(w) = d\#_b(w)\}$$

4. (30 points)

Definition Let $w \in \Sigma^*$. Then ISAAC(w) is the set of words that can be formed by removing any set of symbols from w. For example

$$\text{ISAAC}(abab) = \{e, a, b, aa, ab, ba, bb, aab, aba, abb, bab, abab\}$$

If L is a langauge (a subset of Σ^*) then

$$\text{ISAAC}(L) = \bigcup_{w \in L} \text{ISAAC}(w).$$

For example if $L = \{abab, bbbb\}$ then

$$\text{ISAAC}(L) = \{e, a, b, aa, ab, ba, bb, aab, aba, abb, bab, bbb, abab, bbbb\}$$

Show that if L is regular then ISAAC(L) is regular.