Homework 6, MORALLY Due March 11

1. (50 points) In this program we will look at primes of the form x^2+5y^2. Send your code to Emily, so that if you get wrong answers, I can give you partial credit.

   (a) (0 points but you will need this later)
   Write a program that will, given p, determines if there exists x, y such that
   $$p = x^2 + 5y^2.$$

   (b) (30 points) For all primes $p \in \mathsf{N}$ such that are $7 \le p \le 1000$ run the above program. Produce a table of primes $p \in \mathsf{N}$, such that $7 \le p \le 1000$, of the following form. You must put the table in your pdf. You can either copy and paste it into your latex doc, or take a screenshot.

p	sum of x^2+5y^2?	x^2+5y^2
7	N	
11	N	
13	N	
17	N	
19	N	
23	N	
29	Y	$3^2+5\times 2^2$

   (c) (20 points) Give a conjecture of the following form:
   Let p be an odd prime such that $p \ge 7$. Then
   p is a sum of the form x^2+5y^2 iff BLANK.

2. (50 points) On the untimed midterm1 you wrote two programs to find, given n, a way to write n as a sum of squares. The first one we call GREEDY the second one we call OPTIMAL.

   (a) (0 points but you need to do this for a later part) Let $f(n)$ be

   $$\max\{\text{GREEDY}(1), \text{GREEDY}(2), \cdots, \text{GREEDY}(n)\}.$$

   Write a program that, given n, computes

   $$f(1), \ldots, f(n).$$

   (b) (0 points but you need to do it for the later parts) Run this program on $n = 1000$.

   (c) (20 points) Make a conjecture about what $f(n)$ looks like. (For example: $f(n)$ is ROUGHLY $\sqrt{n}$.)

   (d) (15 points) Let $X_1 = \{n : n \equiv 7 \pmod 8\}$. On the untimed midterm1, problem 1g, you probably found that, from your data,

   $$n \in X_1 \Rightarrow \text{OPTIMAL}(n) = 4.$$

   Find an infinite set X_2 such that $X_1 \cap X_2 = \emptyset$ and, according to your data,

   $$n \in X_2 \Rightarrow \text{OPTIMAL}(n) = 4.$$

   (e) (15 points) Make a conjecture about exactly which numbers n have $\text{OPTIMAL}(n) = 4$.