**Homework 2, MORALLY Due Feb 12**

1. (28 points-7 points each)

   (a) Consider the formula

   $$(x_{11} \vee x_{12}) \wedge (x_{21} \vee x_{22}) \wedge (x_{31} \vee x_{32}) \wedge (x_{41} \vee x_{42}).$$

   How many satisfying assignments does this formula have? Justify!

   (b) Let $n \in \mathsf{N}$ with $n \geq 3$. Consider the formula

   $$(x_{11} \vee x_{12}) \wedge (x_{21} \vee x_{22}) \wedge \cdots \wedge (x_{n1} \vee x_{n2}).$$

   How many satisfying assignments does this formula have? Justify! (Note that it may be a function of $n$.)

   (c) Consider the formula

   $$(x_1 \vee \neg x_2) \wedge (x_2 \vee \neg x_3) \wedge (x_3 \vee \neg x_4) \wedge (x_4 \vee \neg x_1).$$

   How many satisfying assignments does this formula have? Justify! (Do not use a truth table.)

   (d) Give an example of a 2CNF formula which uses 4 variables and is NOT satisfiable.

2. (20 points) Let $n \in \mathsf{N}$. Two formulas $\phi_1$ and $\phi_2$ are *$n$-Equiv* if their truth tables DIFFER on exactly $n$ rows. Note that logical equivalence is 0-Equiv.

   (a) (10 points) Give an example of two formulas $\phi_1, \phi_2$, each on 3 variables, that are 1-Equiv. (Hint: This is easier if you make them in DNF form.)

   (b) (10 points) Give an example of three formulas $\phi_1, \phi_2, \phi_3$, each on 3 variables, such that the following hold:

      i. $\phi_1$ and $\phi_2$ are 1-Equiv.

      ii. $\phi_2$ and $\phi_3$ are 1-Equiv.

      iii. $\phi_1$ and $\phi_3$ are 2-Equiv.

3. (20 points – 5 points each) For each of the following statements write the negation without using any negations signs.

   (a) $x \neq 4$

   (b) $(x_1 \leq x_2) \wedge (x_1 \leq x_3)$

   (c) $(x \leq 5) \vee (x \geq 15)$

   (d) $x < y < z$

4. (32 points-8 points each) You are designing an algorithm for CNFSAT. I will incompletely describe some short cuts you can take. Fill in the BLANK

   The input is of the form

   $C_1 \wedge \cdots \wedge C_m$

   where each $C_i$ is an OR of literals (a literal is a var or its negation).

   (a) If $C_1 = (x_3)$ then you can do $BLANK_1$.

   (b) If $x_4$ appears in the formula but $\neg x_4$ never appears then you can do $BLANK_2$.

   (c) If $C_2 = (x_8)$ and $C_3 = (x_9)$ and $C_4 = (\neg x_8 \vee \neg x_9)$ then you can do $BLANK_3$.

   (d) If $C_4 = (x_{10} \vee \neg x_{11} \vee x_{12} \vee \neg x_{12})$ then you can do $BLANK_4$.

5. (0 point- For FUN and just EMAIL Bill your answer) Contradiction in real life: There are songs where what people *think* the song is about is in contradiction to what the song is really about if you listen to they lyrics carefully. Bill had a blog post on this (Bill has been a blogger on complexity theory since 2008 or so) here)

   READ the blog post.

   EMAIL Bill your thoughts on the post- which song you particular liked, something you learned, some other song that has that property, whatever you want.