CMSC 250 — Assigned: June 20, 2017

Homework 5

Due: June 23, 2017

This assignment is due at 12PM on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit all your answers should be carefully justified.

Each solution must be written independently by yourself - no collaboration is allowed.

In your answers, you may use results that we have proved in lecture, results from discussion sessions, and results from previous homeworks as building blocks for your solutions. We also allow you to assume the results for the parity of sums and products (i.e. you can assume odd + odd = even, even $\times$ odd = even, etc.) Everything else you will have to prove on your own!

Remember that your solutions must be typeset in LaTeX. You can find a template for this homework on the course site. Make sure that each problem (by question number) is separated from each other (i.e. you should not have the solutions to Question 1 and Question 2 on the same page).

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1. [**10 pts**] As a way to boost their group's social dynamic, the Four Myopic Mice decide to put their phones in the middle of the table whilst eating meals, so that no one is tempted to use their phone. At the end of the meal, they all try to get their phone back. However, since they have poor vision, they cannot tell whose phone is whose. Because of this, they just end up taking whichever phone they reach first. In other words, there can be a situation where Mice 1 gets Mice 2's phone, Mice 2 gets Mice 3's phone, Mice 3 gets Mice 4's phone, and Mice 4 gets Mice 1's phone, or a situation where Mice 1 gets Mice 2's phone, Mice 2 gets Mice 3's phone, Mice 3 gets Mice 1's phone, and Mice 4 gets its own phone.

(a) In how many ways can this happen, such that no mouse ends up get their own phone back?

(b) Suppose that there are now 10 myopic mice participating in this. How many ways can it happen now?

2. [**10 pts**] A starfish has one sock and one shoe for each of its 5 arms (each arm has its own corresponding sock and shoe).

In order to be ready to leave, the starfish must do three separate actions on each of its arms: put on the sock, put on the shoe, and tie the shoe lace. Note that the actions for each arm must be completed in order, i.e. for any particular arm, the shoe cannot be put on before the sock, and the shoe must be tied after putting on the sock and shoe, etc. However, the starfish can work on one arm, and then switch to work on another arm. For example, the starfish can put on the sock for the first arm, then the sock for the second arm, and then return to put the shoe onto the first arm.

In how many different ways can the starfish get ready to leave?

3. [**10 pts**] After a pretty successful life financially, you decide to give away your fortune to your five children. Your fortune amounts to $\$100,000,000$. You want to divide this amount in chunks of $\$1,000,000$ and give them to your children in such a way that every child gets at least $\$1,000,000$. Thus the amount that each child gets is a multiple of $1,000,000$. In how many ways can you distribute your wealth among your five children, provided each one receives at least $\$1,000,000$?

4. [**10 pts**] A traveling salesperson needs to visit 4 locations, each of them three times. How many ways can he do this if he's not allowed to start and end in the same location?

5. [**10 pts**] How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 30$, where x_1, x_2, x_3, x_4, x_5 are non-negative integers, given the following restrictions. Four of the variables must be odd, and the remaining variable (which may be odd or even) yields a remainder of 2 when divided by 3? Justify your answer.

6. [**10 pts**] Prove that for any prime numbers a, b, and c, $a^2 - b^2 \neq c^2$.