CMSC 250 Assigned: June 9, 2017

Homework 3

Due: June 13, 2017

This assignment is due at 12PM on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit all your answers should be carefully justified.

Each solution must be written independently by yourself - no collaboration is allowed.

In your answers, you may use results that we have proved in lecture, results from discussion sessions, and results from previous homeworks as building blocks for your solutions. We also allow you to assume the results for the parity of sums and products (i.e. you can assume odd + odd = even, even × odd = even, etc.) Everything else you will have to prove on your own!

Remember that your solutions must be typeset in LaTeX. You can find a template for this homework on the course site. Make sure that each problem (by question number) is separated from each other (i.e. you should not have the solutions to Question 1 and Question 2 on the same page).

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1. **[10 pts]** It's been a long term for Captain Sparrow. He's had his ups and downs running the Black Pearl, but he's loved the experience all the same.

   As the sun sets, Sparrow strolls up to the ship deck and looks at the sea, reflecting on his memories as captain of the Black Pearl. As he gazes out into the waves, he puzzles over a problem that Elizabeth mentioned to him long ago. Specifically, Elizabeth talked about a set (called A), and a relation on A (called R) that is irreflexive, symmetric and transitive.

   (a) Elizabeth insists that the relation $S = (A \times A) \setminus R$ is an equivalence relation. However, Sparrow is a bit skeptical of Elizabeth's claim. Can you prove or disprove Elizabeth's statement?

(b) Sparrow also decides to make a conjecture of his own: that if a relation T on A is irreflexive, anti-symmetric, and transitive, then $U = (A \times A) \setminus T$ is an equivalence relation. Is Sparrow correct? Since Sparrow is very experienced with CMSC 250 problems, you'll have to convince him with either a proof or a counter-example.

2. [10 pts] Answer the following counting problems on relations and functions:

(a) How many reflexive relations are there on a set A, where $|A| = n$?

(b) Let A and B be finite sets of size a and b, respectively. How many distinct functions are there from A to B?

3. [10 pts] Matt is the mayor of the town in which you live. He, like most other people, despises the DMV and is planning on replacing it with his own institution. He is running into one small problem, he wants to make sure that there are enough license plates for every car in town. Matt declares, a license plate string can consist only of the 26 uppercase letters, the 10 digits, and a space character. A "word" is a non-empty sequence of letters and numbers. Each license plate string is either one word (with no spaces) or two words separated by a space character (Please note that the space character cannot begin nor end the license plate string). Furthermore, each license plate string must contain exactly 8 distinct characters (including the space character if there are two words). For example, "CMSC250" is not a valid license plate string, but "CMSZ 250" and "BIGCARSZ" are.

How many license plate strings are possible?

4. **[10 pts]** Scott is preparing to get dressed to attend an important event for the evening. However, he has

quite a full wardrobe, and needs help figuring out how many choices he has before he can pick the perfect one.

He has three choices of tie: polka-dot, striped, and solid-color.

The rest of his wardrobe consists of shirts and pants from n different brands.

For each brand, he has five different colors of shirts.

Scott also has a certain number of pants for each brand. Unfortunately, he does not remember the specific number of pants for each brand, only that he has at least one pair per brand, and that the total number of pairs of pants in his wardrobe is 1000.

Since Scott is very fashionable, he has a very specific definition of what is considered a "complete" outfit. For Scott, a complete outfit consists of a choice of one of his ties, and also a shirt and pair of pants that come from the same brand. Given all of this, can you help

Scott figure out how many different complete outfits he could wear this evening?

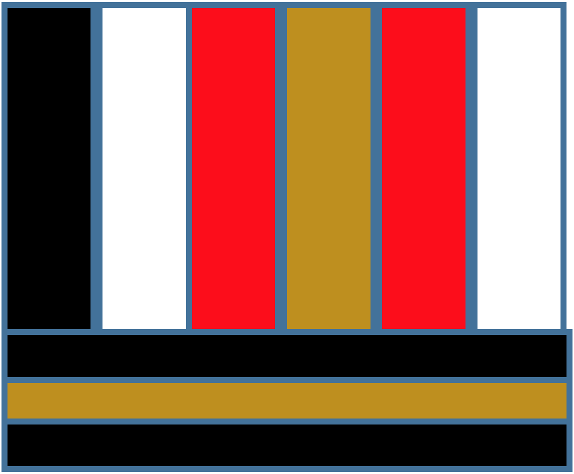
You should be able to determine an exact integer solution.

5. [10 pts] After tasting the power that comes with being the head TA of the Discrete Math class, Varun decided an office job was not for him and began his career as a pirate, joining a large crew off the west coast. Using his superior intellect, armed with counting and probability skills, he quickly navigated his way up the chain of command. In 3 months and 14 days, he became captain. Seeking even more power, Varun renamed his ship the Ruby Perl and sought acceptance into the Large Wooden Pirate Ship Association (LWPSA). Given his speedy rise to power and the confidence radiating from him, the other Pirate Lords had but one task for him: design a suitable Flag for the Ruby Perl, worthy of the LWPSA.

The standards of the flag require 6 vertical stripes placed atop of 3 horizontal stripes (as presented in the example flag). The stripes can be colored black, grey, white, red, and gold. A color may be used multiple times, but two stripes of the same color cannot share a long edge.

Varun looks to you pondering this crucial choice and asks how many possible flags could he make that meet the LWPSA standard?

As an example, here is one such possible flag:



6. [10 pts] April and Joe are packing away their dorm after a long semester. April has n distinct items that she wants to put away: her guitar, her alarm clock, her cup, etc. She could pack them all into a single box if she wanted to, pack each of them into their own box, or group some items together to pack into different boxes.

While pondering how she could go about this, Joe comes over. After talking, they realize that they are facing the exact same problem of packing their items away, with one difference: Joe has $n+1$ distinct items, instead of n.

Perennially wanting to one up his friends, Joe claims that he can pack his items into boxes in more distinct ways than April can. Unconvinced, April asks Joe to prove it. After scratching his head awkwardly for 5 minutes, Joe can't figure it out, and ask you, a student of CMSC 250, to help him out.

Can you help him? In other words, prove that there are more distinct ways to place $n + 1$ items into boxes than n items into boxes.

Note that two ways are considered the same iff for each of the boxes in one of the ways, there is a box in the other way that has the exact same items in it. Further note that there cannot be any empty boxes (cause that would be a waste of boxes!) and each item has to go in a box.