Discussion Session 5
July 5, 2017

1. You draw 5 cards from a standard 52 card deck uniformly at random. What is the probability of you drawing the following hands?

   (a) Four of a kind

   (b) Full House

   (c) Two pairs

   (d) Straight (not a straight flush)

   (e) Flush (not a straight flush)

2. Let a "natural" set be a set where for any "natural" set S, $\forall x \in S, x \in \mathbb{N}$. Prove, using normal (not strong) induction, that if the maximum element in a "natural" set B is n, then the power set of B has at most 2^{n+1} elements.

3. When three dice are rolled what is the probability that one of the dice results in 4?

   Solve it both by determining the probability of the complement event, and the Inclusion Exclusion Formula.

4. Let us play a game of Russian roulette. We are playing with styrofoam guns since we are non-violent. Here is the barrel of the gun, six chambers, all empty. Now watch me as I put two styrofoam bullets into the barrel, into two adjacent chambers. I close the barrel and spin it. I point the gun to you and pull the trigger but nothing happens. Now I am going to pull the trigger one more time. Which would you prefer: that I spin the barrel first or that I just pull the trigger?

5. You run into a town with 100 robots. You know that 99 of these robots tell the truth half the time and lie the other half. You know that in addition, there is 1 constantly truthful robot in town. You take a robot at random and ask him your question seven times, and he tells the truth every time. What is the probability this is the truthful robot?

6. Varun and his crew of pirates are in a battle! After many hours, they have the opportunity to finish off the enemy ship with just one more cannonball. With the enemy ship on their starboard side they must rely on one of their two starboard side gunners, Shawn and Charlie, to land the finishing blow.

Shawn is the more skilled of the two gunners and hits his mark with probability 0.9! Charlie on the other hand is less competent and has an accuracy of just 0.6.

Furthermore, Shawn is also the faster working cannoneer and fires 3 times as many shots as Charlie. This means that the probability that a cannonball fired from the starboard side was fired by Shawn is 0.75.

In these final decisive moments, the last shot successfully sinks their target! What is the probability that Shawn was the one who fired the winning blow?

7. There are 5 yellow balls in bag 1.

   (a) One ball is transferred to bag 2 which contains an unknown number of green balls.

   (b) Bag 2 is then shaken and a ball is selected at random and transferred to bag 1 without seeing its colour.

   (c) Bag 1 is then shaken and a ball is selected at random and transferred to bag 2 without seeing its colour.

   (d) Finally, a ball is chosen at random from bag 2.

   If I tell you before you carry out this process that there's a $\dfrac{3}{5}$ chance you will end up with a green ball at the end, can you work out how many green balls are in bag 2 at the start?

8. Two cards are sequentially drawn (without replacement) from a well-shuffled deck of 52 cards. Let A be the event that the two cards drawn have the same value (e.g. both 4s) and let B be the event that the first card drawn is an ace. Are these events independent?

9. Alice and Bob play a series of games until one of them wins 3 games. Alice and Bob each have probability $\frac{1}{2}$ of winning each game, and no game ends in a draw. When one of the players wins 3 games, what is the probability that the other player has won exactly 1 game?