Discussion Session 4

June 28, 2017

1. (a) Let us select an 8-letter string, where each letter is selected from the 26 letters of the alphabet, independent from each other and uniformly at random. What is the probability that we select a string that contains 3,4, or 5 vowels?

   (b) Suppose instead we are selecting 8-letter strings where all of the letters are distinct, uniformly at random.

   Now what is the probability that we select a string that contains 3,4, or 5 vowels?

2. Prove that $\forall n \in \mathbb{N}$, $2^{2n} - 1$ is a multiple of 3.

3. $2n$ dots are placed around the outside of the circle. n of them are colored red and the remaining n are colored blue. Going around the circle clockwise, you keep a count of how many red and blue dots you have passed. If at all times the number of red dots you have passed is at least the number of blue dots, you consider it a successful trip around the circle. Prove that no matter how the dots are colored red and blue, it is possible to have a successful trip around the circle if you start at the correct point.

Assume that $n \in \mathbb{Z}^+$.

4. Consider the Fibonacci numbers F_i, where $F_1 = F_2 = 1$ and $F_{j+2} = F_j + F_{j+1}$, for $j \geq 1$. Prove that for all integers $n \geq 1$,

$$\sum_{i=1}^{n} F_i^2 = F_n F_{n+1}$$