CMSC 250: Discrete Structures

Summer 2017

Discussion Session 2

June 14, 2017

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1. A "super-duper-mega-p-cool solution" is a solution that consists of finite sets X, Y and functions $f : X \to Y$ and $g : Y \to X$ such that *all* of the following conditions are simultaneously satisfied:

- *The super condition:* $g \circ f$ is a bijection
- *The duper condition:* $g \circ f$ is not the identity function
- *The mega condition:* f is not a surjection
- *The p condition:* g is not an injection
- *The cool condition:* X has exactly 3 elements

(a) Give us an example of a "super-duper-mega-p-cool solution". Make sure you define X, Y, f

and g clearly, either using set-builder notation or by explicitly writing out the elements of the sets.

(b) Justify why your solution is “super” (show that it satisfies the “super” condition).

(c) Justify why your solution is “duper”.

(d) Justify why your solution is “mega”.

(e) Justify why your solution is “p”.

(f) Justify why your solution is “cool’.

2. If $g : C \to D$ is a function, we can define how the function maps a subset of C in the following way:

$$\text{If } S \subseteq C, \text{ then define } g(S) = \{g(s) : s \in S\}$$

Now suppose we have some invertible function $f : X \to Y$. Prove that $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$, where $A, B \subseteq Y$.

3. In CombinatoriaLand, all newborns are given a name of exactly 10 characters, which can be upper case letters or lower case letters.

(a) Given this, how many unique names are possible in CombinatoriaLand?

(b) How many of these names are composed solely of vowels?

(c) How many of these names are composed solely of consonants?

(d) The benevolent dictator of CombinatoriaLand implements a new law, called the Law of Large Names. Under the new law, all names must contain at least one vowel and one consonant in them. Given this, how many unique names are now possible in CombinatoriaLand?

For this question, consider "y" to be a consonant.

4. How many 3-digit numbers have distinct non-zero digits?

Generalize your answer for a k-digit number, where $k \leq 9$.

5. A palindrome is a word that reads the same forward and backward. For example, abba and cdededc are palindromes. How many palindromes of length n can be formed using 26 lower-case letters of the English alphabet?

6. Your favorite pizza place in the world, Big Nick's Pizzeria, is known for its variety of different pizzas. Big Nick has 5 different kinds of tomato sauces and 6 different kinds of cheese. In addition, he can add one of any 20 different toppings, which are optional. On top of all of these choices, you can choose a thin crust or a thick crust. How many different pizzas can you possibly order from Big Nick's? What if Big Nick can add any number of toppings (i.e. not just one)?

7. How many 5-digit numbers can I construct using the numbers $\{1, 2, 3, 4, 5\}$ without repeat, such that the number is divisible by 2, 4, and 8?

8. You have 3 cats: Felice, Leo, and Cheater. You wish to place each one of them into one of 8 boxes for a nap. Each of the boxes conveniently labeled from 1 to 8, such that they are distinguishable.

(a) You know that the cats don't play well together when napping, so you don't want to place more than 1 cat in any box. Given this, how many ways can you place the cats into the boxes?

(b) After a restful nap, the cats wake up and run off to play with balls of yarn in the corner. After such a successful nap, you decide that from now on, a pair of cats are allowed to nap together in the same box (but not all of them together, just in case they start scheming their escape together!). Given this, how many ways can you place the cats in boxes in their next nap session?