CMSC 250: Discrete Structures

Summer 2017

Discussion Session 1

June 7, 2017

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1. Show that $p \vee q \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

2. Show that $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$.

3. Prove the following: The sum of two integers is even iff their difference is even.

4. Let $A = \{n \mid n = 2k + 5 \text{ for some } k \in \mathbb{N}\}$ and $B = \{n \mid n = 2j + 1 \text{ for some } j \in \mathbb{N}\}$. Is $A \subseteq B$?

5. Let $A = \{n \in \mathbb{N} \mid n = 2k^2 - 3, \text{ for some } k \in \mathbb{N}\}$ and $B = \{n \in \mathbb{N} \mid n = j^2 + 3 \text{ for some } j \in \mathbb{N}\}$. Prove that $A \not\subseteq B$.

6. Let $A = \{n \in \mathbb{N} \mid n \geq 2 \text{ and } n = 4j - 5, \text{ for some } j \in \mathbb{N}\}$ and $B = \{n \in \mathbb{N} \mid n \geq 0 \text{ and } n = 2k + 1 \text{ for some } k \in \mathbb{N}\}$. Prove that $A \subset B$.

7. Let x be an integer. If $x > 1$, then $x^3 + 1$ is composite.

8. Show that at least three of any 25 days chosen must fall in the same month of the year.

9. Recall the *cartesian product* of A and B, denoted by $A \times B$, is the set of all ordered pairs formed by taking an element from A together with an element from B in all possible ways. That is, $A \times B = \{(a, b) \mid a \in A, b \in B\}$. Prove that if A and B are non-empty sets then $A \times B = B \times A$ iff $A = B$.

10. For any integer $n \geq 2$, prove that if no prime $p \leq \sqrt{n}$ divides n, then n must be prime.