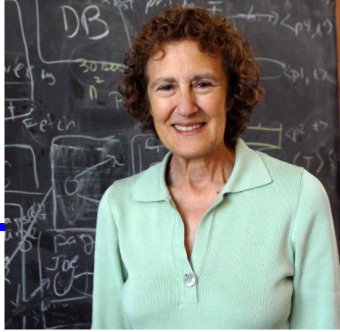
CMSC 330: Organization of Programming Languages

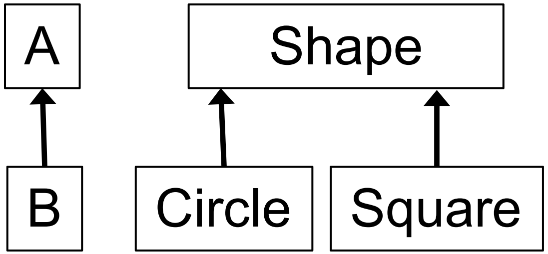
Subtyping

Subtyping



- The Liskov Substitution Principle:
  - Let `P(x)` be a property provable about objects x of type T. Then `P(y)` should be true for objects y of type S where S is a subtype of T.
- In other words
  - If S is a subtype of T, then an S can be used anywhere a T is expected
- Commonly used in object-oriented programming
  - Subclasses can be used where superclasses expected .
  - This is a kind of *polymorphism*

What is subtyping?



- Sometimes "*every B is an A*"
  - Example:
    - Every Circle or Square is a Shape
- Subtyping expresses this
  - "*B is a subtype of A*" means: "every object that satisfies the rules for a B also satisfies the rules for an A"
- Goal: code written using A's specification operates correctly even if given a B
  - Plus: clarify design, share tests, (sometimes) share code

Subtyping

- A type S is a **subtype** of T, written S <: T, when any term of type S can safely be used in a context where a term of type T is expected.

- S <: T means
  - S is more informative than T.
  - the values of type S are a subset of the values of type T.

The Subsumption Rule

$$\frac{G \vdash \texttt{e:S} \quad \texttt{S <: T}}{G \vdash \texttt{e:T}} \quad \text{(T-Sub)}$$

- This rule tells us that, if S <: T, then every element t of S is also an element of T.

- For example, if we define the subtype relation so that

$$G \vdash \texttt{\{x:Int, y:Int\} <: \{x:Int\}}$$

then we can use the subsumption rule to derive

$$G \vdash \texttt{\{x=0,y=1\} <: \{x:Int\}}$$

which is what we need to make our motivating example typecheck.

Subtyping: A Preorder

- The subtype relation is **formalized as a collection of inference rules** for deriving statements of the form **`S <: T`**, pronounced "S is a subtype of T" (or "T is a supertype of S").

- The subtype relation should always be a **preorder**, meaning that it is reflexive and transitive.

Reflexivity: $\mathtt{S} <: \mathtt{S}$ (S-REFL)

Transitivity:
$$\frac{\mathtt{S} <: \mathtt{U} \qquad \mathtt{U} <: \mathtt{T}}{\mathtt{S} <: \mathtt{T}} \quad \text{(S-TRANS)}$$

Subtyping — Records: Width Subtyping

- Width Subtyping:

$$\{\mathtt{l}_i : \mathtt{T}_i \; ^{i \in 1..n+k}\} <: \{\mathtt{l}_i : \mathtt{T}_i \; ^{i \in 1..n}\} \qquad \text{S-RCDWIDTH}$$

- A **longer record** constitutes a more demanding—i.e., more informative—specification, and so describes a **smaller set** of values.

- Examples:
  - `{x:Int, y:Int} <: {x:Int}`
  - `{x:Int, y:Int, z:Bool} <: {x:Int}`

Quiz

```
{x:Int, y:Int} <: {y:Int}
```

A. True
B. False

Quiz

```
{x:Int, y:Int} <: {y:Int}
```

A. True
B. False

Subtyping — Records: Depth Subtyping

- Depth Subtyping:

$$\frac{\text{for each } i \quad \mathtt{S}_i <: \mathtt{T}_i}{\{\mathtt{l}_i : \mathtt{S}_i{}^{\,i \in 1..n}\} <: \{\mathtt{l}_i : \mathtt{T}_i{}^{\,i \in 1..n}\}} \qquad \text{S-RCDDEPTH}$$

- It is safe to allow the types of **individual fields** to vary, as long as the types of each corresponding field in the two records are in the subtype relation.
- Example:
  - `{x:{a:Int, b:Int}, y:{m:Int}} <: {x:{a:Int},y:{}}`

Quiz

Which is the subtype of

```
{ x:{a:Int, b:Bool} }
```

```
A. {a:Int,b:Bool}
B. {x:{a:Int}}
C. {x:{a:Int}, y:{b:Bool}}
D. {x:{a:Int, b:Bool,c:Int}, y:{d:Int}}
```

Quiz

Which is the subtype of

```
{x:{a:Int,b:Bool}}
```

```
A. {a:Int,b:Bool}
B. {x:{a:Int}}
C. {x:{a:Int}, y:{b:Bool}}
D. {x:{a:Int, b:Bool,c:Int}, y:{d:Int}}
```

Subtyping Derivations

$$
\dfrac{\dfrac{}{\texttt{\{a:Nat,b:Nat\}} <: \texttt{\{a:Nat\}}}\ \textsc{S-RcdWidth} \qquad \dfrac{}{\texttt{\{m:Nat\}} <: \texttt{\{\}}}\ \textsc{S-RcdWidth}}{\texttt{\{x:\{a:Nat,b:Nat\},y:\{m:Nat\}\}} <: \texttt{\{x:\{a:Nat\},y:\{\}\}}}\ \textsc{S-RcdDepth}
$$

Subtyping — Records: Permutation Subtyping

- Permutation Subtyping: the order of fields in a record does not make any difference to how we can safely use it

$$\frac{\{\mathtt{k}_j : \mathtt{S}_j\ ^{j \in 1..n}\} \text{ is a permutation of } \{\mathtt{l}_i : \mathtt{T}_i\ ^{i \in 1..n}\}}{\{\mathtt{k}_j : \mathtt{S}_j\ ^{j \in 1..n}\} <: \{\mathtt{l}_i : \mathtt{T}_i\ ^{i \in 1..n}\}} \quad \text{S-RCDPERM}$$

- Example:
  - **`{c:Unit,b:Bool,a:Int} <: {a:Int,b:Bool,c:Unit}`**
  - **`{a:Nat,b:Bool,c:Unit} <: {c:Unit,b:Bool,a:Nat}`**

Quiz

Which rules will we need to build a derivation of the following?

```
{x:Int,y:Int,z:Int} <: {y:Int}
```

A. S-RCDDEPTH
B. S-RCDWIDTH
C. S-RCDPERM
D. S-TRANS

Quiz

Which rules will we need to build a derivation of the following?

```
{x:Int, y:Int, z:Int} <: {y:Int}
```

A. S-RCDDEPTH
B. S-RCDWIDTH
C. S-RCDPERM
D. S-TRANS

Subtyping — Functions

- Functions can be passed as arguments to other functions, we must also give a subtyping rule for function types

$$\frac{T_1 <: S_1 \qquad S_2 <: T_2}{S_1 \rightarrow S_2 <: T_1 \rightarrow T_2} \qquad \text{S-ARROW}$$

- Notice that the sense of the subtype relation is **reversed** (contravariant) for the **argument types** in the left-hand premise, while it runs in the **same direction** (covariant) for the **result types** as for the function types themselves.

Subtyping — Functions

- **Intuition**
  - Let’s say I have a Java function, f, which takes a Cat object and returns an Animal. What are the subtypes of this function? Well, if it takes a Cat then I can certainly replace this function with one that takes an Animal. Likewise, if it returns an Animal then I can certainly replace this function with one that returns a Cat (or Dog). Therefore, I conclude that…

```
(Animal → Cat) <: (Cat → Animal)
(Animal → Dog) <: (Cat → Animal)
```