Logistic Regression

CMSC 422

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Slides partially adapted from MARINE CARPUAT

Today’s topics

- Continue Discussion on Logistic Regression

- Gradient Descent for LR

- Multilabel Classifications

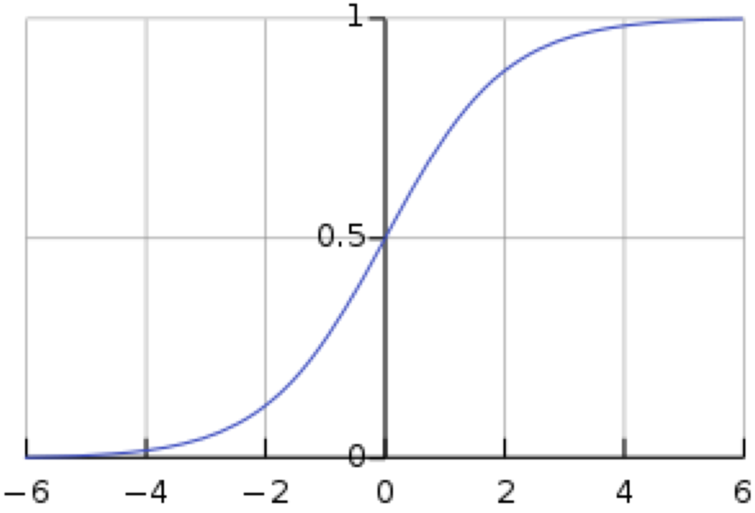
- HW4 posted, Programming assignment due date extended to March 25

Logistic Regression

- Binary classification

$$P(Y^{(i)} = 1 | X^{(i)}, \theta) = g(< \theta, X^{(i)} >)$$

$$P(Y^{(i)} = 0 | X^{(i)}, \theta) = 1 - g(< \theta, X^{(i)} >)$$



Sigmoid function

$$g(z) = \frac{1}{1 + \exp(-z)}$$

Logistic Regression

- Maximum Likelihood

$$\max_{\theta} \quad \prod_{i=1}^{N} P(Y^{(i)} | X^{(i)}, \theta)$$

⬇

$$\max_{\theta} \quad \prod_{i=1}^{N} g(<\theta, X^{(i)}>)^{Y^{(i)}} (1 - g(<\theta, X^{(i)}>))^{1-Y^{(i)}}$$

⬇

$$\max_{\theta} \quad \sum_{i=1}^{N} \underbrace{Y^{(i)} \log g(<\theta, X^{(i)}>) + (1 - Y^{(i)}) \log(1 - g(<\theta, X^{(i)}>))}_{\text{Cross-entropy loss function}}$$

How to solve it?

- Gradient Descent

- A good property of sigmoid:

$$\nabla_z g(z) = g(z)(1 - g(z))$$

- SGD: $\theta_{k+1} = \theta_k + \eta(Y^i - g(< \theta, X^i >))X^{(i)}$

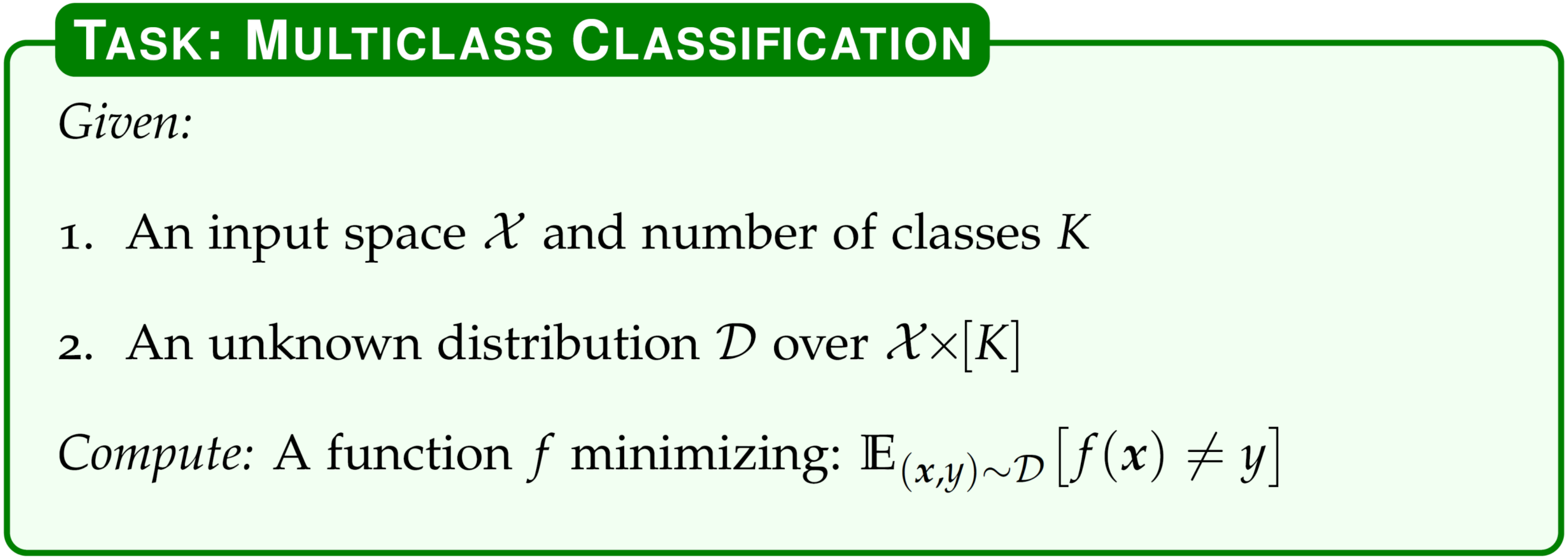
- Why? Intuition behind the updates

Multiclass classification

- Real world problems often have multiple classes (text, speech, image, biological sequences…)

- How can we perform multiclass classification?
  - Straightforward with decision trees or KNN
  - Can we use the perceptron algorithm?

Reductions for Multiclass Classification



TASK: MULTICLASS CLASSIFICATION

Given:

1. An input space $\mathcal{X}$ and number of classes K

2. An unknown distribution $\mathcal{D}$ over $\mathcal{X} \times [K]$

Compute: A function f minimizing: $\mathbb{E}_{(\boldsymbol{x},y)\sim\mathcal{D}}\left[f(\boldsymbol{x}) \neq y\right]$

Task: Binary Classification

Given:

1. An input space $\mathcal{X}$

2. An unknown distribution $\mathcal{D}$ over $\mathcal{X} \times \{-1, +1\}$

Compute: A function f minimizing: $\mathbb{E}_{(\boldsymbol{x},y)\sim\mathcal{D}}\left[f(\boldsymbol{x}) \neq y\right]$

How many classes can we handle in practice?

- In most tasks, number of classes K < 100
- For much larger K
  - we need to frame the problem differently
  - e.g, machine translation or automatic speech recognition

Reduction 1: OVA

- "One versus all" (aka "one versus rest")
    - Train K-many binary classifiers
    - classifier k predicts whether an example belong to class k or not
    - At test time,
        - If only one classifier predicts positive, predict that class
        - Break ties randomly

Algorithm 12 OneVersusAllTrain($\mathbf{D}^{multiclass}$, BinaryTrain)

```
for $i = 1$ to $K$ do
    $\mathbf{D}^{bin} \leftarrow$ relabel $\mathbf{D}^{multiclass}$ so class $i$ is positive and $\neg i$ is negative
    $f_i \leftarrow$ BinaryTrain($\mathbf{D}^{bin}$)
end for
return $f_1, \dots, f_K$
```

Algorithm 13 OneVersusAllTest($f_1, \dots, f_K, \hat{\boldsymbol{x}}$)

```
$score \leftarrow \langle 0, 0, \dots, 0 \rangle$                // initialize $K$-many scores to zero
for $i = 1$ to $K$ do
    $y \leftarrow f_i(\hat{\boldsymbol{x}})$
    $score_i \leftarrow score_i + y$
end for
return $\operatorname{argmax}_k score_k$
```

Time complexity

- Suppose you have N training examples, in K classes. How long does it take to train an OVA classifier
  - if the base binary classifier takes O(N) time to learn?
  - if the base binary classifier takes O(N^2) time to learn?

Reduction 2: AVA

- All versus all (aka all pairs)

- How many binary classifiers does this require?

Algorithm 14 AllVersusAllTrain($\mathbf{D}^{multiclass}$, BinaryTrain)

```
$f_{ij} \leftarrow \emptyset, \forall 1 \leq i < j \leq K$
for $i$ = 1 to $K$-1 do
    $\mathbf{D}^{pos} \leftarrow$ all $\boldsymbol{x} \in \mathbf{D}^{multiclass}$ labeled $i$
    for $j$ = $i$+1 to $K$ do
        $\mathbf{D}^{neg} \leftarrow$ all $\boldsymbol{x} \in \mathbf{D}^{multiclass}$ labeled $j$
        $\mathbf{D}^{bin} \leftarrow \{(\boldsymbol{x}, +1) : \boldsymbol{x} \in \mathbf{D}^{\mathrm{pos}}\} \cup \{(\boldsymbol{x}, -1) : \boldsymbol{x} \in \mathbf{D}^{\mathrm{neg}}\}$
        $f_{ij} \leftarrow$ BinaryTrain($\mathbf{D}^{bin}$)
    end for
end for
return all $f_{ij}$s
```

Algorithm 15 AllVersusAllTest(all f_{ij}, $\hat{\boldsymbol{x}}$)

```
$score \leftarrow \langle 0, 0, \ldots, 0 \rangle$                    // initialize $K$-many scores to zero
for $i$ = 1 to $K$-1 do
    for $j$ = $i$+1 to $K$ do
        $y \leftarrow f_{ij}(\hat{\boldsymbol{x}})$
        $score_i \leftarrow score_i + y$
        $score_j \leftarrow score_j - y$
    end for
end for
return argmax$_k$ $score_k$
```

Time complexity

- Suppose you have N training examples, in K classes. How long does it take to train an AVA classifier
  - if the base binary classifier takes O(N) time to learn?
  - if the base binary classifier takes O(N^2) time to learn?