CMSC 330 Quiz 5 Spring 2022 Solutions

Q1. Lambda Calculus

Consider the following lambda expression.

```
λa. λb. b c λc. d f a
```

Note: To represent λ, you may either copy and paste the symbol **λ** or just type the characters **L** or **** in your solutions.

Q1.1. Make the parenthesis explicit

```
(λa. (λb. ((b c) (λc. ((d f) a)))))
```

Q1.2. Which of the following are free (or unbound) variables? Select all that apply.

- a
- b
- **c**
- **d**
- **f**

Q1.3. Which of the following are valid α-conversions? Select all that apply.

- **λx. λb. b c λc. d f x**
- λw. λb. b c λc. d f a
- λa. λb. b x λx. d f a
- **λa. λb. b c λw. d f a**
- **λa. λy. y c λc. d f a**

Q2. Concepts

For each of the questions below, select whether the given statement is true or false.

Q2.1. Reducing lambda expressions with call-by-name and call-by-value always yields the same result, assuming that all expressions terminate. **T**/F

Q2.2. Reducing lambda expressions with call-by-name and call-by-value always takes the same number of steps/reductions. T/**F**

Q3. Beta Reduction

Reduce the following lambda calculus expression to the β-normal form.

```
(λy. λy. y y) a (λy. y) b
```

Show each step, including any β-reduction or α-conversion. If there is infinite recursion, write "Infinite Recursion".

Notes:

- You must make all parenthesis explicit before reducing the expression.
- You also must perform valid α-conversions to remove all ambiguity/duplicate variables.
- To represent λ, you may either copy and paste the symbol λ or just type the characters L or \ in your solutions.

```
(λy. λy. y y) a (λy. y) b
= ((((λy. (λy. (y y))) a) (λy. y)) b)     --- Explicit Parenthesis
= ((((λy. (λm. (m m))) a) (λn. n)) b)     --- α-Conversion
= (((λm. (m m)) (λn. n)) b)
= (((λn. n) (λn. n)) b)
= ((λn. n) b)
= b
```

Q4. Mystery Operator

Suppose we have a mystery lambda expression `mys` such that for any input x, a, we have the following:

```
mys x a = x
```

Note: To represent λ, you may either copy and paste the symbol λ or just type the characters **L** or **** in your solutions.

Q4.1. Give a possible lambda expression for `mys`.

```
λx. λy. x
```

Answers with form `λa. λb. x` will not receive partial credit.

Q4.2. Using the expression from **Q4.1**, reduce the following expression to the β-normal form.

```
mys (λx. x)
```

Show each step, including any β-reduction or α-conversion. If there is infinite recursion, write "Infinite Recursion".

```
mys (λx. x)

= (λx. λy. x) (λx. x)

= (λy. (λx. x))
```

Even if expression in Q4.1 was incorrect, we have given partial credit for correct β-reduction.