CMSC330 Spring 2019 Midterm 2
11:00am / 12:15pm / 2:00pm

Solution

Name (PRINT YOUR NAME as it appears on gradescope):

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Discussion Time (circle one) **10am 11am 12pm 1pm 2pm 3pm**

Instructions

- Do not start this test until you are told to do so!
- You have 75 minutes to take this midterm.
- This exam has a total of 100 points, so allocate 45 seconds for each point.
- This is a closed book exam. No notes or other aids are allowed.
- Answer essay questions concisely in 2-3 sentences. Longer answers are not needed.
- For partial credit, show all of your work and clearly indicate your answers.
- Write neatly. Credit cannot be given for illegible answers.

	Problem	Score
1	PL Concepts	/13
2	Finite Automata	/31
3	Context Free Grammars	/17
4	Parsing	/16
5	Operational Semantics	/10
6	Lambda Calculus	/13
	Total	/100

1. PL concepts [13 pts]

A) [5 pts] Circle true or false for each of the following 5 questions (1 point each)

True / ***False***	In OCaml, if an exception is thrown, then the executing program will terminate
True / False	OCaml variables are immutable
True / ***False***	If x and y are aliases, changing the content in the location referenced by x will cause it to no longer be an alias of y
True / False	If a lambda calculus expression reduces to a beta-normal form using call-by-value order, then it will also do so using call-by-name
True / False	You can create a cyclic data structure in OCaml (i.e., one that points to itself)

B) [4 pts] Consider the following OCaml definitions for `f`, `g`, and `h` (each is a `int -> int` function).

```
let f z =
  let y = ref 0 in
  y := !y + z;
  !y
```

```
let g =
  let x = ref 1 in
  (fun z ->
    x := !x + 1;
    !x+z)
```

```
let h =
  (fun z -> let x = z+1 in
    let _ = (print_int z,print_int x) in
    0)
```

	Answer:
Which of these functions is not *referentially transparent*?	***either g or h***
Which function's execution outcome *depends on OCaml's evaluation order*	***h***
What is a *side effect* carried out by at least one of the functions?	***Printing or incrementing***
Which function's execution is *only* interesting/useful because of its side effects, not what it returns?	***h***

C) [4 pts] Check the box next to each function that is *tail recursive* (they all type check and run properly).

☐
```
let rec sum lst =
  match lst with
    [] -> 0
  | h::t-> h + sum t
```

☑
```
let rec max lst r =
  match lst with
    [] -> r
  | h::t ->
      if r>h then max t r
              else max t h
```

☑
```
let rec pow2 x =
  if x = 1 then true
  else
    let y = x/2 in
    if y*2 = x then pow2 y
    else false
```

☐
```
let rec prod lst =
  match lst with
    [] -> 1
  | h::t -> (prod t) * h
```

2. Finite Automata [31 pts]

A) [4 pts] Circle true or false for each of the following 4 questions (1 point each)

True / ***False*** NFAs are more expressive than DFAs (i.e., they can describe more languages)

True / ***False*** Every CFG has an equivalent NFA

True / ***False*** Every formal language has a unique DFA that generates it

True / ***False*** Regexes are more expressive (can generate more languages) than DFAs

B) [6 pts] For each of the following statements, check the DFA box if it's true for DFAs, and the NFA box for NFAs. You may check neither or both boxes.

☐DFA ☑NFA Can transition to multiple states at once with a symbol

☐DFA ☑NFA Can have epsilon transitions

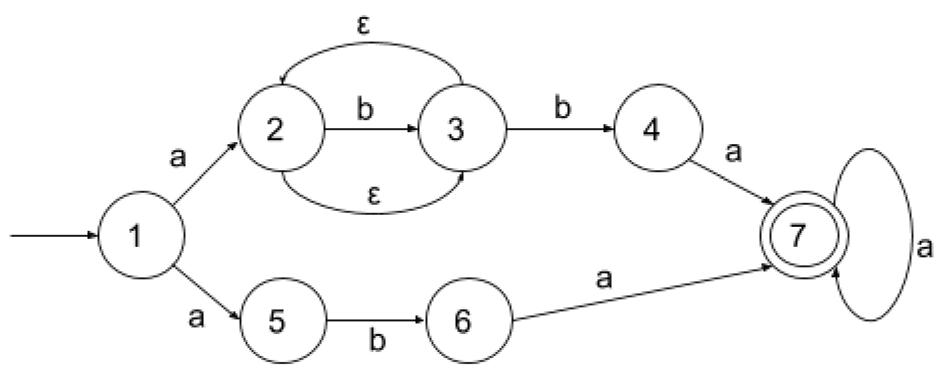
☑DFA ☑NFA Can have multiple final states

☐DFA ☐NFA Always has at least one final state

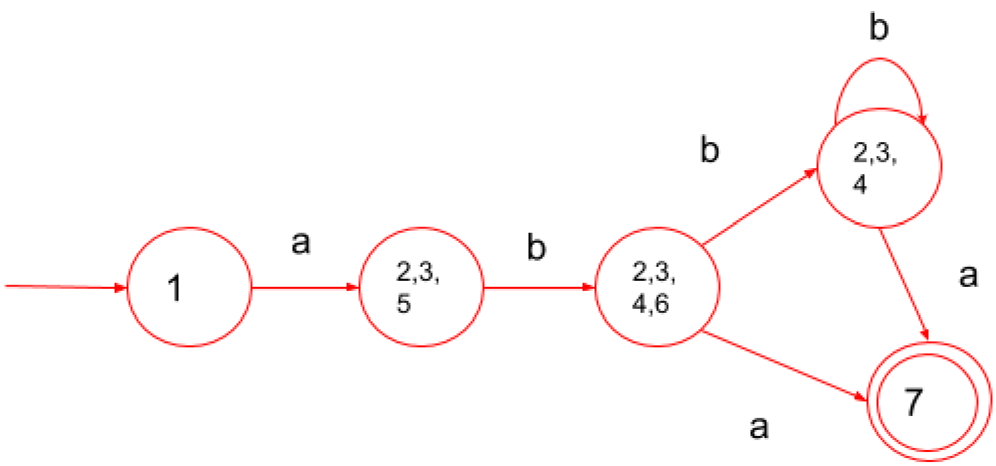
☐DFA ☑NFA Easy to translate directly from a regular expression

☑DFA ☑NFA Can accept an empty string

C) [6 pts] Draw a DFA that is equivalent to the following NFA.



Solution:

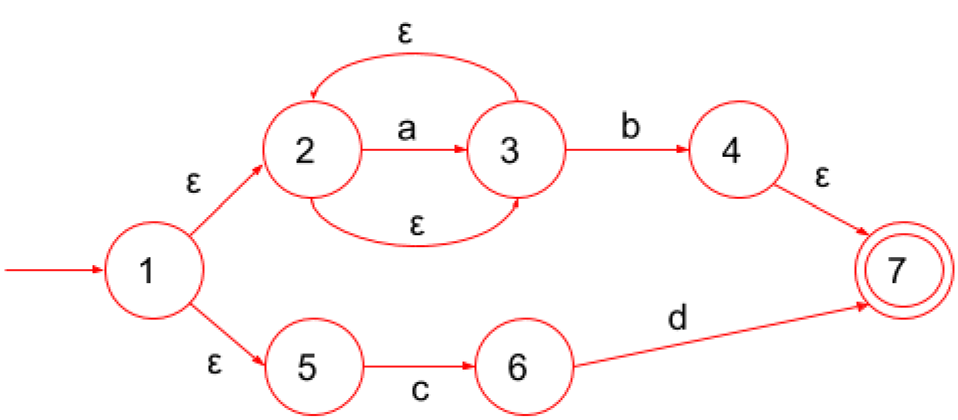


D) [4 pts] Circle any of the following strings that would be accepted by the nfa from the previous problem.

aba ***abbbbba*** **aa** ***abaa***

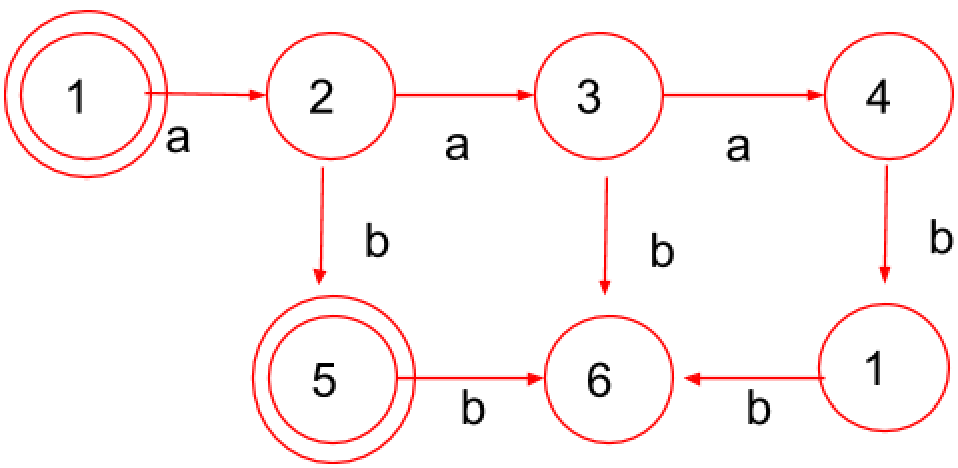
E) [6 pts] Draw an NFA that accepts the same language as the regex (**a*b**)|(**cd**). Here are some examples this NFA will accept: **b**, **ab**, **cd**, **aab**, **aaaaab**

Solution:



F) [5 pts] Draw a DFA that accepts strings of the form $\mathbf{a}^n\mathbf{b}^n$ where $0 \le n \le 3$ over Σ = { **a**,**b** }

Solution:



3. Context Free Grammars [17 pts]

A) [4 pts] Check the box next to the strings that are accepted by the following CFG. Note that here and below all nonterminals are in italics (like *T* and *W*) and terminals are in bold (like **a**, **b**).

$T \rightarrow \mathbf{a}W \mid \mathbf{b}$
$W \rightarrow \mathbf{b} \mid \mathbf{b}T \mid \mathbf{a}W$

☐ **abba** ☑ **aaabb** ☐ **baa** ☑ **aab**

B) [5 pts] Create a CFG for the language of all strings of the form $n^x f^z a^y$ where $x \geq y \geq 0$ and $z > 0$. Example strings in the language are **nfa**, **f**, **nnnfaa**. Example strings *not* in the language are **a**, **n**, **fa**, **nfaa**.

Solution:

S -> nSa | nS | A
A -> fA | f

C) [4 pts] Rewrite the following grammar so that it can be parsed by a recursive descent parser. Note that parentheses and commas, below, are terminals (along with **r**, **u**, and **o**).

S → *A***)**
A → *A***,r** | *A***,u** | **(o**

Solution:

S → A)
A → (oB
B → ,rB | ,uB | ε

D) [4 pts] The following CFG is ambiguous. Rewrite the grammar to remove the ambiguity. Note that minus sign is a terminal (along with **1**, **2**, and **3**).

E → *E* **-** *E* | *N*
N → **1** | **2** | **3**

Solution:

E → N - E | N
N → 1 | 2 | 3

4. Parsing and Scanning [16 pts]

A) [3 pts] Recall the scanner for SmallC. Suppose, when you tokenize the variable "`for2`", your tokenizer returned `[Tok_ID("for");Tok_Int(2)]` instead of `[Tok_ID("for2")]`. How would you fix this? (Write 1-2 sentences only.)

Solution:
The issue here lies with the Tok_ID regular expression, as we know from the project that IDs can contain digits, but this ID ignores digits when it is tokenized. Therefore, we have to change the ID regex to include digits ([a-zA-Z][a-zA-Z0-9]*).

B) [5 pts] Consider the following CFG. Compute the first sets for each nonterminal.

$S \rightarrow mB \mid aA$
$A \rightarrow cS \mid \varepsilon$
$B \rightarrow 1\#S \mid dB \mid St \mid Ao$

FIRST(S) = ***{m, a}***

FIRST(A) = ***{c, ε}***

FIRST(B) = ***{1, d, m, a, c, o}***

C) [8 pts] Complete the implementation for a recursive-descent parser for the provided CFG, given on the next page. Write your answer on the next page.

(scratch space, do not write your final answer here)

```
exception ParseError of string

let tok_list = ref [];;

let match_tok x = match !tok_list with
|(h::t) when x = h -> tok_list := t
|_ -> raise (ParseError "bad match")

let lookahead () = match !tok_list with
|[] -> None
|(h::t) -> Some h
```

$S \rightarrow mB \mid aA$

$A \rightarrow cS \mid \varepsilon$

$B \rightarrow 1\#S \mid dB \mid St \mid Ao$

```
let rec Parse_S() =
      if lookahead() = Some “m” then
            (match_tok “m”; Parse_B())
      else (* fill-in below *)
      if lookahead() = Some “a” then
            (match_tok “a”; Parse_A())
      else
            raise(Parse Error “not valid input”)


and Parse_A() =
      if lookahead() = Some “c” then (* fill-in below *)
            (match_tok “c”; parse_S())
      else
            ()


and Parse_B() =
      if lookahead() = Some “1” then
            (match_tok “1”; match_tok “#”; parse_S())
      else (* fill-in below *)
      if lookahead() = Some “d” then
            (match_tok “d”; Parse_B())
      else if lookahead() = Some “m” || lookahead = Some “a” then
            (parse_S(); match_tok”t”)
      else if lookahead() = Some “c” || lookahead() = Some “o” then
            (parse_A(); match_tok “o”)
      else
            raise(Parse Error “not valid input”)
```

5. Operational Semantics [10 pts]

A) [5 pts] Using the rules given below, show: `let x = 1 in 1 + x ⟶ 2`

In the rules, *e* refers to an expression whose abstract syntax tree (AST) is defined by the following grammar, where *x* is an arbitrary identifier and *n* is an integer.

> *v* ::= *n*
> *e* ::= *x* | *v* | let x = *e* in *e* | *e* + *e*

$$\text{Id}\,\frac{A(x) = v}{A; x \longrightarrow v} \qquad \text{Int}\,\frac{}{A; n \longrightarrow n}$$

$$\text{Let}\,\frac{A; e1 \longrightarrow v1 \quad A, x : v1; e2 \longrightarrow v2}{A; \text{let } x = e1 \text{ in } e2 \longrightarrow v2} \qquad \text{Add}\,\frac{A; e1 \longrightarrow v1 \quad A; e2 \longrightarrow v2 \quad v3 \text{ is } v1 + v2}{A; e1 + e2 \longrightarrow v3}$$

Solution:

$$\textit{Let}\,\frac{\textit{Int}\,\dfrac{}{A; 1 \longrightarrow 1} \qquad \textit{Add}\,\dfrac{\textit{Int}\,\dfrac{}{A, x : 1; 1 \longrightarrow 1} \quad \textit{Id}\,\dfrac{A, x : 1(x) = 1}{A, x : 1; x \longrightarrow 1} \quad \textit{2 is 1 + 1}}{A, x : 1; 1 + x \longrightarrow 2}}{A; \textbf{\textit{let}}\ x = 1\ \textbf{\textit{in}}\ 1 + x \longrightarrow 2}$$

B) [5 pts] Below are operational semantics rules for a simple language, where the abstract syntax tree for expressions e and values *v* defined as follows.

v ::= false | true
e ::= v | not *e* | if *e1* then *e2*

$$\text{true}\,\frac{}{true \longrightarrow true} \qquad \text{false}\,\frac{}{false \longrightarrow false} \qquad \text{nottrue}\,\frac{e \rightarrow true}{not\ \ e \longrightarrow false} \qquad \text{notfalse}\,\frac{e \rightarrow false}{not\ \ e \longrightarrow true}$$

$$\text{Iftrue}\,\frac{\begin{array}{c} e1 \rightarrow true \\ e2 \longrightarrow v \end{array}}{if\ \ e1\ \ then\ \ e2 \longrightarrow v} \qquad \text{Iffalse}\,\frac{e1 \rightarrow false}{if\ \ e1\ \ then\ \ e2 \longrightarrow true}$$

Write a function `eval` of type `exp -> exp`, where `exp` is the OCaml representation of *e*:

```
type exp =
    Tru                          (* corresponds to true *)
  | Fals                         (* corresponds to false *)
  | If of exp * exp              (* corresponds to if e1 then e2 *)
  | Not of exp                   (* corresponds to not e *)
```

The `eval` function evaluates an expression in a manner consistent with the rules. For example:

```
eval(Tru) = Tru
eval(Not (Not Tru)) = Tru
```

etc.

```
let rec eval e =
  match e with
  | Tru -> Tru
   (* FILL IN REST *)
  | Fals -> Fals
  | If (e1, e2) -> if (eval e1) = Tru then
                       (eval e2)
                     else
                       Tru
  | Not e' -> if (eval e') = Tru then
                 Fals
               else
                 Tru
```

6. Lambda Calculus [13 pts]

A) [2 pts] Circle the **free variables** in the following λ-term:

`λx. y (λz.z y x) z`

B) [2 pts] Write a lambda calculus term that is *α-equivalent* to the one above.

Solution:
Examples: ***λx. y (λz.z y x) z***
λa. y (λb.b y a) z

C) [4 pts] Circle true or false for the following questions (1 point each)

True / False	The beta-normal form of `(λx.y z) z` is `y z`
True / False	The fixpoint combinator Y is used in lambda calculus to achieve recursion
True / ***False***	A *Church numeral* is the encoding of a real number as a lambda calculus term
True / ***False***	The expression `(λx. y) z` encodes `let x = y in z`

D) [5 pts] Reduce the following lambda expressions into beta-normal form. Show each beta reduction. If already in normal form or infinite reduction, write "normal form" or"infinite reduction", respectively.

1) `(λx. (λy. y x) (λz. x z)) (λy. y y)`

```
=> (λ x. (λ z. x z) x) (λy. y y)
=> (λ x. x x) (λ y. y y)
=> (λ y. y y) (λ y. y y)
Infinite reduction
```

2) `(λx. x y z) (λy. z)`

```
=> (λy. z) y z
=> z z
```