Lecture 08/29/17

Lecturer: Xiaodi Wu

Reading Assignment: Course Website; [AB] Chap 0.
Welcome to CMSC 652: Complexity Theory
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Welcome to the new academic year!
Instructor

- Instructor: Prof. Xiaodi Wu
- Contact: AVW 3257, xwu@cs.umd.edu
- Research: Quantum Information and Computation
- Joint Center for Quantum Information and Computer Science (QuICS)
Teaching Team

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- **NO Quantum** covered in the lecture. Check CMSC 858K by Andrew Childs. You are welcome to ask quantum questions!
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▶ CMSC 457: Introduction to Quantum Computation (Spring 18)
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► CMSC 457: Introduction to Quantum Computation (Spring 18)

TA

► Sheng Yang, styang@cs.umd.edu
Question

What is this course? Why are you here?
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- the study of the computation itself in an abstract way: theory of computation.
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- complexity theory studies the power of computation in terms of consumed computational resources.
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CMSC 652 is about:

- the study of the computation itself in an abstract way: theory
  of computation.
- complexity theory studies the power of computation in terms
  of consumed computational resources.
- it can be deemed as the opposite side of algorithms.
Theoretical Computer Science

Computer Science

Mathematics

Statistics

Social Science

Core ToC

Comp. Comp.

Structure, Insights, Interconnectivity,…

Property testing

Verification

Distributed

Lower bounds

Communication Complexity

Quantum Computing

Crypto

Learning

AGT

Logic

Randomness

Mathematics
Intuitive questions to address

▶ Are algorithms that are given more time always able to solve more problems?
Complexity Theory

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► Are algorithms that are given more time always able to solve more problems?
► Is verifying solutions to problems easier than coming up with such solutions?
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- Can tossing coins help us compute faster?
Complexity Theory

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▶ Are algorithms that are given more time always able to solve more problems?
▶ Is verifying solutions to problems easier than coming up with such solutions?
▶ Can tossing coins help us compute faster?
▶ Can we define randomness?
▶ Is finding approximate answers easier than finding exact answers?
▶ Can we prove that some interesting problems cannot be solved efficiently?
▶ Can you verify that an algorithm solves a problem without solving it yourself?
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Complexity Theory: Methodology

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The power of computation in terms of consumed computational resources such as time, memory, communication, number of rounds of communication, and randomness.
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Necessary Steps:
- Abstraction and modeling of the computation.
Complexity Theory: Methodology

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The power of computation in terms of **consumed computational resources** such as *time, memory, communication, number of rounds of communication, and randomness*. **quantum**.

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The power of computation in terms of consumed computational resources such as time, memory, communication, number of rounds of communication, and randomness. quantum.

Necessary Steps:

- Abstraction and modeling of the computation.
- Modelling of different computational resources.
- Measure of the consumed resources.
- Comparison of the power of computation.
Methodology

We address all these questions using **rigorous mathematical** tools.
Complexity Theory: Methodology

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Expectation
▶ You have ”mathematical maturity” (e.g., are comfortable with proofs and abstract reasoning).
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Expectation
- You have "mathematical maturity" (e.g., are comfortable with proofs and abstract reasoning).
- You are interested in the material.
Complexity Theory: Methodology

Methodology
We address all these questions using **rigorous mathematical** tools.

Expectation

- You have "mathematical maturity" (e.g., are comfortable with proofs and abstract reasoning).
- You are interested in the material.
- You are willing to spend time outside of class in order to better understand the material presented in class.
Complexity Theory: Teaching Philosophy

- Emphasize more on the conceptual messages!
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Complexity Theory: Teaching Philosophy

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What if I want to do research in this direction ...
Further references will be provided! You are always welcome to ask questions!
More logistics

Office Hours

- Wu: Tu Th 3:30 pm - 4:30 pm at AVW 3257, or by appointments.
- Yang: W 2:30 pm - 3:30 pm at AVW 3164.
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Websites

- Course website: syllabus, reading assignments, handouts, and so on. Check Frequently!!.
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- **Piazza**: announcements, discussion forum, ask for helps.
- **ELMS**: distribute and submit assignments, grades, solutions.
Languages

\( L_f = \{ x \in \{0, 1\}^* : f(x) = 1 \} \) for languages or decision problems.

Example

\[
\text{INDSET} = \{ < G, k > : \exists S \subset V(G) \text{ s.t. } |S| \geq k \\
\text{and } \forall u, v \in S, \overline{uv} \notin E(G) \}.
\]
3.1 Asymptotic notation

- **(b)** If there exist positive constants $n_0$, $c_1$, and $c_2$ such that at and to the right of $n_0$, the value of $f(n)$ always lies between $c_1 g(n)$ and $c_2 g(n)$ inclusive.

- **(c)** $f(n)$ belongs to the set $\Theta(g(n))$ if there exist positive constants $c_1$ and $c_2$ such that it can be "sandwiched" between $c_1 g(n)$ and $c_2 g(n)$, for sufficiently large $n$. Because $\Theta(g(n))$ is a set, we could write "$f(n) \in \Theta(g(n))" to indicate that $f(n)$ is a member of $\Theta(g(n))$. Instead, we will usually write "$f(n) = \Theta(g(n))" to express the same notion. You might be confused because we abuse equality in this way, but we shall see later in this section that doing so has its advantages.

- For all values of $n$ at and to the right of $n_0$, the function $f(n)$ lies at or above $c_1 g(n)$ and at or below $c_2 g(n)$. In other words, for all $n / n_0$, the function $f(n)$ is equal to $g(n)$ to within a constant factor. We say that $g(n)$ is an asymptotically tight bound for $f(n)$.

- The definition of $\Theta(g(n))$ requires that every member $f(n) \in \Theta(g(n))$ be asymptotically nonnegative, that is, that $f(n)$ be nonnegative whenever $n$ is sufficiently large. (An asymptotically positive function is one that is positive for all sufficiently large $n$.) Consequently, the function $g(n)$ itself must be asymptotically nonnegative, or else the set $\Theta(g(n))$ is empty. We shall therefore assume that every function used within $\Theta$-notation is asymptotically nonnegative. This assumption holds for the other asymptotic notations defined in this chapter as well.