The following exercises are designed to test your understanding of recursion. The functions are defined using a variant of LISP known as meta-LISP. In order to aid your understanding, the function defined in problem 1 is identical to the one below:

\[
\text{drop}(x) = \begin{cases}
\text{null } x & \Rightarrow \text{nil} \\
\text{car } x \text{ cons drop(cdr } x) & \text{else}
\end{cases}
\]

The idea is that
- \( a \times = \text{car } x \)
- \( d \times = \text{cdr } x \)
- \( n \times = \text{null } x \)
- \( a.t = \text{atom } x \)
- \( a.b = a \text{ cons } b \)
- \( <a> = a \text{ cons nil } = \text{a list whose single element is } a \)
- \( a*b = \text{concatenate lists } a \text{ and } b \text{ (i.e. append list } b \text{ to list } a) \)
- \( \text{reverse}[x] = \text{reverses the top level list } x \). For example \( \text{reverse}[(A B C)] = (C B A) \). But \( \text{reverse}(((A B C)(D E))) = ((D E)(A B C)) \).

1. Consider the function \( \text{drop} \) defined by
   \[\text{drop}[x] \leftarrow \begin{cases}
   \text{null } x & \Rightarrow \text{nil} \\
   \text{else } [a \times].\text{drop} [d \times].
   \end{cases}\]
   Compute (by hand) \( \text{drop}[(A B C)] \). What does \( \text{drop} \) do to lists in general?

2. What does the function
   \[\text{r2}[x] \leftarrow \begin{cases}
   \text{null } x & \Rightarrow \text{nil} \\
   \text{else } \text{reverse}[a \times].\text{r2}[d \times].
   \end{cases}\]
do to lists of lists? How about
   \[\text{r3}[x] \leftarrow \begin{cases}
   \text{null } x \text{ else } \text{r3}[a \times].\text{r4}[d \times].
   \end{cases}\]

3. Compare the following function with the function \( \text{r3} \) of the preceding example:
   \[\text{r3'}[x] \leftarrow \begin{cases}
   \text{null } x \text{ else } \text{r3'}[d \times].\text{r3'}[a \times].
   \end{cases}\]

4. Consider \( \text{r5} \) defined by
   \[\text{r5}[x] \leftarrow \begin{cases}
   \text{null } x \text{ or } \text{null } d \times & \Rightarrow x \\
   \text{else } [a \times].\text{r5}[d \times].\text{r5}[a \times].\text{r5}[d \times].\text{r5}[d \times].
   \end{cases}\]
 Compute \( \text{r5}[(A B C D)] \). What does \( \text{r5} \) do in general. Needless to say, this is not a good way of computing this function even though it involves no auxiliary functions.