1. In Dijkstra’s algorithm, at each iteration the nodes \( v \) in outside that are adjacent to the node \( u \) being processed, may get new, potential shortest path distances. In other words, the array \( d \) is possibly updated for those vertices.

Draw a directed, weighted graph \( G = (V, E) \) on four vertices such that at each iteration all of the nodes in outside get new, potential shortest path distances. In other words, the array \( d \) is updated for every vertex in outside. To keep it simple, \( G \) should have no cycles.

2. Let \( G = (V, E) \) be a directed, weighted graph with weight function \( w : E \rightarrow \{0, 1, 2, \ldots, s\} \) for some nonnegative integer \( s \).
   
   (a) Modify Dijkstra’s algorithm to compute the shortest paths from a given source vertex \( a \) in time \( O(m + sn) \).

   (b) Modify your algorithm from Part (a) to run in time \( O((m + n) \log s) \). Hint: How many distinct shortest-path potential distances can there be in outside at any given point in time?

3. The Generalized Eulerian Cycle problem is given a directed graph \( G = (V, E) \) with \( n \) nodes, \( m \) edges numbered 1 to \( m \), and a list of \( m \) non-negative integers \( k_1, k_2, \ldots, k_m \), where \( 0 \leq k_i \leq n \), is there a cycle that crosses edge \( e_i \) exactly \( k_i \) times (for \( 1 \leq i \leq m \))?

   Show that the Generalized Eulerian Cycle problem is in NP. Make sure to state what the certificate is, and argue that the verification is polynomial time.

4. CHALLENGE PROBLEM (will not be graded): Generalize Problem (1) to an arbitrary number of vertices \( n \).